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PRACTICAL APPLICATION OF THE CRITERION FOR DETERMINING LONG-TERM STRENGTH AND RESIDUAL DURABILITY OF STRUCTURAL ELEMENTS AT LOW TEMPERATURE CREEP

Matviyiv Y.Ya., Andrushchak I.E, Ganulich B.K., Kradinova T.A. Practical application of the criterion for determining the long-term strength and residual durability of structural elements for low-temperature creep. In the article, in practice, the previously formulated criterion for determining the long-term strength and residual longevity of structural elements with cracks in the case of a long static load is considered and its application to the examples of problems for beams of an open profile during their prolonged stretching and bending.

Key words: long-term strength, residual durability, open profile beam, low-temperature creep, period up to critical crack growth, stress intensity factor.

Матвіїв Ю.Я., Андрущак І.Є., Гануліч Б.К., Крадінова Т.А. Практичне застосування критерію для визначення довготривалої міцності та залишкової довговічності елементів конструкцій за низькотемпературної повзучості. У статті на практиці розглянуто сформульваний раніше критерій для визначення довготривалої міцності та залишкової довговічності елементів конструкцій з тріщинами за довготривалого статичного навантаження та застосування його на прикладах задач для балок відкритого профілю за їх довготривалого розтягу та згину.

Ключові слова: довготривала міцність, залишкова довговічність, балка відкритого профілю, низькотемпературна повзучість, період докритичного росту тріщини, коефіцієнт інтенсивності напружень.

Матвиив Ю.Я., Андрущак И.Е., Ганулич Б.К., Крадинова Т.А. Практическое применение критерия для определения длительной прочности и остаточной долговечности элементов конструкций при низкотемпературной ползучести. В статье на практике рассмотрены сформулированный ранее критерий для определения длительной прочности и остаточной долговечности элементов конструкций с трещинами при длительной статической нагрузке и применение его на примерах задач для балок открытого профиля при их длительном растяжении и изгибе.

Ключевые слова: длительная прочность, остаточная долговечность, балка открытого профиля, низкотемпературная ползучесть, период до критического роста трещины, коэффициент интенсивности напряжений.

For the technical diagnostics of metal structures, as well as reinforced concrete engineering structures, where elements are used for long-term static loads, the establishment of periods between preventive examinations and defectoscopic control, and hence the determination of the need for their repair or replacement, and thus their reliable operation, are necessary reliable methods for predicting long-term durability and durability, especially in the presence of cracks. Exploring this problem is purely empirical, through experimental research, is technically rather difficult and not always possible in principle. Thus there is a need to create a reliable propagation theory and describe the processes of origin and distribution of low-temperature creep cracks, as well as methods for calculating long-term durability and residual resource of said structural elements.

The issue of long-term durability of thin-walled structural elements is currently devoted to many works, especially with experimental studies [1–4]. However, most of them consider defect-free elements of structures. In the works [5-6], based on the first law of thermodynamics for the delayed destruction of thin-walled structural elements for long-term cyclic and static loads, an energetic approach for estimating the period of subcritical growth of cracks for such loads is formulated. We apply this approach to the calculation of long-term strength of thin-walled elements of structures with cracks of low temperature creep.

Formulation of the problem. Consider a thin-walled element of a construction made of quasi-crimped material. Linear parameters b_i characterize the configuration of the element, and the power parameter p is the external load parameter applied at a reduced temperature when the plastic volumes of the loaded body are low temperature creep [2]. We believe that the methods of defectoscopy have established the absence in the element of the design of large (in comparison with its size) defects that do not exceed a certain value $2l_0$. The task is to determine the smallest value of the external load, during which the given time $t = t_*$ there will be no destruction of this element.

Similarly [7, 8], we assume that in the vicinity of the most intense point of this element there is a crack with a characteristic size $2l_0$. Such a defect can be a rectilinear fracture with the greatest length

$2l_0$. The problem can be solved by the proposed method and in the case of other defects (cavity, inclusion, etc.), but only defects of the type of cracks that are most dangerous are considered here.

By methods of the theory of elasticity we determine the main stresses σ_1, σ_2 at the point O of this element, considering it initially as defectless. We'll get

$$\sigma_1 = f_1(p, b_i), \quad \sigma_2 = f_2(p, b_i), \quad \eta_0 = \frac{f_2(p, b_i)}{f_1(p, b_i)}; \tag{1}$$

where $f_j(p, b_i)$ $(j = 1, 2)$ are well-defined functions.

Given the continuity of the stress tensor, and also the fact that the value l_0 is small, we will assume (while increasing only the margin of safety) that is in the vicinity of the point G with the smallest diameter D ($D >> 2l_0$) around the point O here is a homogeneous stressed state with major stresses σ_1, σ_2. Assume now that in the vicinity of the point O of this element there is a rectilinear length fraction $2l_0$ the most dangerous orientation relative to the direction of the main stresses σ_1, σ_2. Since $D >> 2l_0$, then, the presence of a crack in the length $2l_0$ of the body will not affect the stressed state on the line oo the vicinity of the point G, that is, there is realized a two-tensile stress tension σ_1, σ_2. Consequently, the stressed state in the vicinity of such a crack can be approximated (it can be shown that the inaccuracy obtained in this case will go to the margin of safety.

Plate with arbitrarily oriented crack. Consider an infinite perfectly elastic-plastic plate with a straight-line macrocrack of initial length $2l_0$, which extends over the infinity of uniformly distributed forces of intensity in mutually perpendicular directions at an angle α to the line of the crack (Fig. 1), as well as for an unlimited plate with such a defect with a two-wise stretching effort σ_1, σ_2. We believe that in the plastic zones near the top of the crack there is a phenomenon of low temperature creep. Determine the parameters of the external load $\sigma_1 = \sigma_{1*}(t_*)$, $\sigma_2 = \sigma_{2*}(t_*)$, in which the residual durability of the plate will not exceed the specified value $t = t_*$.

Since this problem is reversed to the problem of determining the period of subcritical growth of a low-temperature creep crack $t_* = t_*(\sigma_{1*}, \sigma_{2*})$, we first consider the direct problem.

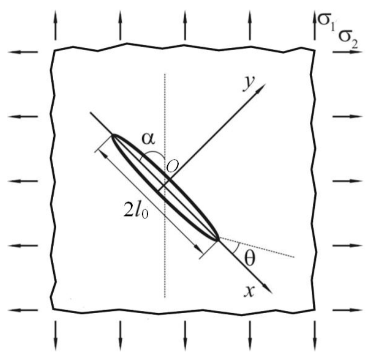

Fig. 1. – Scheme load of the plate with an arbitrarily oriented crack

As shown in [7, 8], the maximum intensity of stress at the vertices of the crack is achieved at $\alpha \approx \pi/2$ at $\eta_0 < 1$ $(\eta_0 = \sigma_1^{-1}\sigma_2)$ and, analogically, $\alpha = 0$, $\eta_0 > 1$. From the results of papers [7, 8] it follows that this corresponds to the maximum value of the rate of propagation of the crack $dl/dt = V_{\max}$. So, for orientation $\alpha = \pi/2$ for $\eta_0 < 1$ and $\alpha = 0$ for $\eta_0 > 1$, the fracture and the least durability of the plate will be most dangerous. Find for these cases $t = t_*$. Based on the results of [7-10] we obtain an equation for determining the period of subcritical growth of a low temperature creep crack:

$$\frac{dl}{dt}=\frac{A_{2t}K_{CC}^{-2m}(K_I^{2m}-K_{thc}^{2m})}{1-K_{CC}^{-2}K_I^2},$$

(2)

under initial and final conditions

$$t=0,\ l(0)=l_0;\quad t=t_*,\ l(t_*)=l_*;\quad K_I(F,l_*)=K_{CC}\,. \tag{3}$$

Here A_{2t},m are the characteristics of low temperature creep; $K_{C\tilde{N}}$ is critical value of $K_{\rm I}$; $K_I(F,l)=F\sqrt{\pi l}$.

Integrating equation (2) under initial and final conditions (3), we obtain

$$t_*=\frac{K_{CC}^{2(m-1)}}{A_{2t}}\int_{l_0}^{l_*}\frac{K_{CC}^2-\pi lF_*^2}{(\pi lF_*^2)^m-K_{thc}^{2m}}dl,\ \ F_*=\begin{cases}\sigma_{*1}, & \alpha=\pi/2,\ \eta_0<1;\\ \sigma_{2*}, & \alpha=0,\ \eta_0>1.\end{cases} \tag{4}$$

Assuming that $l_*>>l_0$, $K_{CC}>>F_*\sqrt{\pi l_0}$, formula (4) can be approximated to represent this way

$$t_*\approx\frac{l_0z}{A_{2t}}\left[\frac{1}{m-1}-\frac{zK_{thc}^{2m}}{K_{CC}^{2m}(2m-1)}\right],\quad z=\left(\frac{K_{C\tilde{N}}^2}{\pi l_0F_*^2}\right)^m \tag{5}$$

From here

$$F_*\approx\frac{K_{CC}}{\sqrt{\pi l_0}}\left\{t_*l_0^{-1}A_{2t}(m-1)\left[1+\frac{t_*A_{2t}K_{thc}^{2m}(m-1)^2}{l_0K_{CC}^{2m}(2m-1)}\right]\right\}^{-\frac{1}{2m}}. \tag{6}$$

On the basis of the relations (4) – (6) for the determination of critical values $\sigma_{1*}=\sigma_{1*}(t_*),\ \sigma_{2*}=\sigma_{2*}(t_*).$, we write the following formulas:

$$\sigma_{1*}=F_*,\ \ \eta_0<1;\quad \sigma_{2*}=F_*,\ \ \eta_0>1, \tag{7}$$

where $\eta_0=\sigma_{1*}\sigma_{2*}^{-1}$.

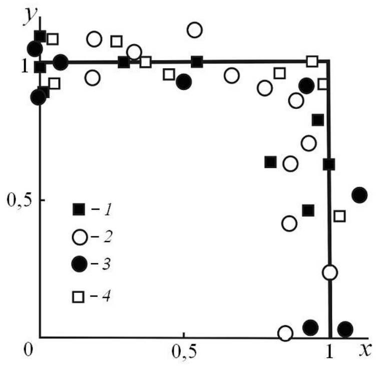

Fig.2. – Diagram of boundary loads for the plate: solid line - ratio (7.8); experiments: *1* – steel [11]; *2–4* – pig-iron samples in different states (*2* – $\sigma_t=345{,}3\ MPa$; *3* – $\sigma_t=185{,}4\ MPa$; *4* – $\sigma_t=228{,}6\ MPa$) [12]

We will make the following replacements in (7) $x=\sigma_{1*}q_*^{-1}$, $y=\sigma_{2*}q_*^{-1}$, де q_* – critical importance extending efforts q for a one-size-fits-all load. Then (7) can be written in this form

$$x=1,\qquad (\alpha=\pi/2,\ \eta_0<1),\qquad y=1,\qquad (\alpha=0,\ \eta_0>1). \tag{8}$$

Based on the dependences (8) in Fig.2, diagrams of boundary loads for a plate with a crack are constructed. Experimental results of low-temperature strength with two-wise stiffness of thin-walled elements without cracks are also presented here [11, 12], which are well in agreement with the results (8). Built on fig. 2 diagram of the boundary loads for plates with cracks and is the basis for calculating the low-temperature strength of thin-walled elements of structures with cracks.

The criterion of long-term residual strength. If the boundary-equilibrium state of the vicinity G is found, then the critical parameter of the external load $p = p_*$ is calculated from the condition

$$f_1\left(p_*, b_i\right) = F_*, \tag{9}$$

which will give the lower (most dangerous with the defect of the material) the value of the marginal load. We write the relation (9) in this form

$$\begin{aligned} &\sigma_{1*} - F_* = 0, \qquad \eta_0 \le 1; \ (\eta_0 = \sigma_{2*}\sigma_{1*}^{-1}) \\ &\sigma_{2*} - F_* = 0, \qquad \eta_0 \ge 1 \end{aligned} \tag{10}$$

which is the equation of the boundary stress diagram in the Cartesian coordinate system $O\sigma_{1*}\sigma_{2*}$ (see also Fig.2). The diagram (10) limits the area of values of the main stresses σ_1, σ_2, safe in relation to the strength of the structural element containing defects of this type during the specified lifetime $t = t_*$ of its operation. Taking into account this, as well as using the relation (10), we obtain the following condition of strength of quasi-violent bodies:

$$\sigma_{1*}\left(p_{i*}\right) - F_*\left(l_0,\, t_*, \eta_0\right) < 0\ , \qquad \left(\eta_0 = \sigma_{1*}^{-1}\sigma_{2*}\right). \tag{11}$$

where the value F_* is determined in the (6).

In engineering practice, cases where initial defects in structural elements are small, but have different configurations (non-linear, surface non-transverse, etc.) are often encountered. To determine the long-term durability of structural elements in such cases, repeat the procedure described above, but for the defects of the given configuration. If the initial defects in the structural elements are not small and are proportional to the dimensions of the structural element, then to determine the residual durability of such elements during a predetermined period of its operation, we solve the direct problem (determination of the residual durability of the structural element with a crack) and the equation thus obtained for t_* (see, for example, (5)) we determine the critical value of the external load.

Determination of the acceptable size of the original defects. Along with the definition of the long-term strength of structural elements with cracks for engineering practice, it is important to establish the admissible dimensions of such defects when their resource and workload are specified. We solve this problem in the following way. We repeat all the considerations that are given at the beginning of this section up to the relation (5). Then from the relation (5), assuming that $K_{thc}^{2m}(\pi l_0 F_*^2)^{-m} << 1$, we determine the approximate permissible value of the size of the initial defect $l_{0*} = l_0(t_*, F_*)$, in which the given resource t_* is provided, i.e.

$$l_{0*} \approx \left(\frac{K_{CC}^2}{\pi F_*^2}\right)^{\frac{m}{m-1}} \left[A_{2t} t_* (m-1)\right]^{\frac{-1}{m-1}}. \tag{12}$$

If the defects a_i in the configuration parameters are not transverse and not straightforward, or large, then to determine their admissible sizes $a_{i*} = a_i(t_*, F_*)$, we solve the direct problem, that is, we construct a formula of type (5), from which we find a_{i*}.

Determination of long-term durability of open profile beams with cracks for their tensile strength. Consider the beams of the open profile of the channel (Fig. 6) and the corner (Fig. 10), which are weakened by cracks and are stretched by the long-term efforts of P. The problem is to find the largest allowable values of effort $P = P_*$ that will provide the residual resource l_0 at given initial sizes of cracks $t = t_*$.

The solution of this problem is carried out similarly to the above. For this we find the relation of type (5) in the case of each profile of the beams. The relations (7.28) and (7.37) for cases of a channel and a corner can be approximated (believe that, $l_* >> l_0$, $K_{CC} >> 1{,}12 P_* \sqrt{\pi l_0}$) to represent this way

$$t_{i*} \approx \frac{1.25 l_0 K_{C\tilde{N}}^{2m}}{A_{2t}(1{,}25\pi l_0 P_{i*}^2 F_i^{-2})^m} \left[\frac{1}{m-1} - \frac{K_{thc}^{2m}}{(1{,}25\pi l_0 P_{i*}^2 F_i^{-2})^m (2m-1)}\right] (i = 1;\ 2.). \tag{13}$$

Here $i=1$ corresponds to the case of a channel, and $i=2$ – to the case of a corner. Solving approximately the equation (13), we obtain the following formula for determining the permissible value of effort $P=P_*$

$$P_* \approx F_i \frac{K_{CC}}{1{,}12\sqrt{\pi l_0}} \left\{ 0{,}8 t_* l_0^{-1} A_{2t}(m-1)\left[1+\frac{t_* A_{2t} K_{thc}^{2m}(m-1)^2}{1{,}25 l_0 K_{CC}^{2m}(2m-1)} \right] \right\}^{-\frac{1}{2m}}. \qquad (14)$$

Assuming that the beams are made of 10HCHF steel (the characteristics of low temperature creep are determined by the relations (7.24)) and the initial length of the cracks is equal to 0.005 m, the relation (14) can be written even more

$$p_{i*} = P_* F_i^{-1} \approx 465{,}2\left\{ 2{,}9 t_* + 18{,}2\cdot 10^{-5} t_*^2 \right\}^{-0.125} \qquad (0 < t_* < 2\cdot 10^4) \qquad (15)$$

on fig.3 according to the formula (15) graphical dependence of the average value p_{i*} of admissible efforts P for the beams of the separation of the channel and the angle on the value of the residual resource t_* is constructed.

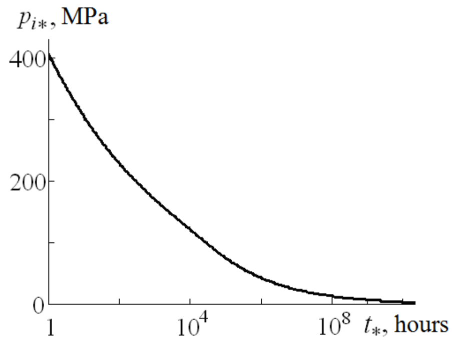

Fig.3. – Dependence of the parameter of the allowable load value p_{i*} from the given residual resource t_*

Estimation of allowable sizes of initial defects in open profile beams for their bending. We believe that the open-beam beams of the brand (Fig.13) and the two-javelin (Fig.15) are weakened by cracks of initial length $l=l_0$ and exposed to long-term (at the time $t=t_*$) bending moments M. The problem is to determine the greatest magnitude of the initial crack $l_0 = l_{0*}$, in which the beams will not be destroyed during the time $t=t_*$ given a given long-term load of moments M.

This task is to some extent reversed to determine the residual resource of such beams. The general approach to an approximate solution to such a problem is presented above. According to this approach, it is necessary to construct a closed solution of an approximate solution of the direct problem as an equation of type (5) and to find an unknown value $l_0 = l_{0*}$ from it. To do this, we will analyze the solutions of the direct tasks for the brand and the two-tauren, which are given in the second paragraph of this section. On the basis of this, and also considering that $l_* >> l_0$, $K_{CC} >> 1{,}12 M W_{xi}^{-1}\sqrt{\pi l_0}$, the solution of a direct problem can be approximated to be written as follows

$$t_{i*} \approx \frac{1{,}25 l_0 K_{C\tilde{N}}^{2m}}{A_{2t}(1{,}25\pi l_0 M^2 M_{xi}^{-2})^m}\left[\frac{1}{m-1} - \frac{K_{thc}^{2m}}{(1{,}25\pi l_0 M^2 W_{xi}^{-2})^m (2m-1)} \right] \ (i=1;\ 2). \qquad (16)$$

Here W_{xi} is a moment of resistance of the cross-section when bending the beam relative to the axis xx if $i=1$ for the brand and if $i=2$ for the two-turret. Then from the relation (16), assuming that $K_{thc}^{2m}(1{,}25\pi l_{0i} M W_{xi}^{-1})^{-m} << 1$, we determine the allowable value of the size of the initial defect $l_{0i} = l_{0i*}$, in which the given resource t_* is provided, that is

$$l_{0i*} = \left(\frac{K_{CC}^2}{\pi M^2 W_{xi}^{-2}} \right)^{\frac{m}{m-1}} \left[A_{2t} t_* (m-1) \right]^{\frac{-1}{m-1}}. \tag{17}$$

For the case where the beams are made of steel 10ХСНД (characteristics of low temperature creep are determined by the relations (7.24)) and the load of beams is equal to $p = MW_{xi}^{-1} = 170$ MPa, the relation (7.17) can be recorded even more

$$l_{0i*} = 4,6 \cdot 10^{-2} \cdot t_*^{-1/3}; \quad (0 < t_* < 10^4). \tag{18}$$

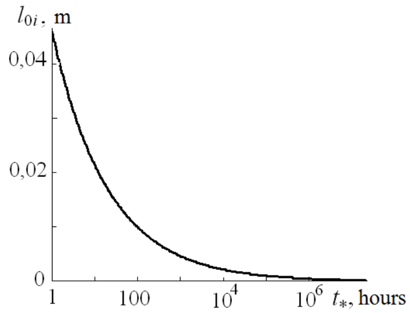

Fig. 4. - Graphical dependence of the allowable value of the size of the initial defect $l_{0i} = l_{0i*}$ on the value of the residual resource t_*

By the formula (18) in Fig.4 graphical dependence of the allowable value of the size of the initial defect $l_{0i} = l_{0i*}$ on the value of the residual resource t_* is constructed.

Conclusions and perspectives of further research. The criterion of long-term strength of elements of structures with cracks for their long-term static load and local low temperature creep is formulated. On the basis of this criterion, the long-term strength of the open-beam beams from the steel 10ХСНД (channel, corner) was determined for their tensile strength and the presence of rectilinear cracks. It also determines the admissible sizes of cracks in the taurus and twin taurus due to their long bending and specified service life. The dependence of long-term durability on the planned resource of beams is analyzed.

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