[illegible] 519.622

[illegible]

[illegible] k-[illegible]

[illegible] k-[illegible]

[illegible] k-[illegible]

[illegible]

$$\frac{dx}{dt} = f\left(x,t\right), \tag{1}$$

[illegible] $f\left(x,t\right)$ - [illegible] x [illegible] t, [illegible]

$$f_i\left(x,t\right) \in C_{x,t}^{(1,0)}\left(D\right), \tag{2}$$

[illegible]

$$D = \left\{0 < t < \infty;\ x \in \mathbb{R}^n\right\}.$$

(2) [illegible] $f\left(x,t\right)$ [illegible] D [illegible] x_i [illegible] x [illegible] $\left\|x\right\| < \delta$ [illegible]

$$f\left(x,t\right) = A\left(t\right)x + \psi\left(x,t\right),$$

[illegible] $$A\left(t\right) = \left(\frac{\partial f\left(x,t\right)}{\partial x}\right)_{|x=0},$$ [illegible] $\psi\left(x,t\right)$ [illegible].

[illegible] $\left\|x\right\| < \delta$ [illegible] (1) [illegible]

$$\frac{dx}{dt} = A\left(t\right)x + \psi\left(x,t\right). \tag{3}$$

[illegible] [1] [illegible] $x = 0$ [illegible]

$$\frac{dx}{dt} = A\left(t\right)\cdot x \tag{4}$$

[illegible] (3), [illegible] (1) [illegible] $\psi\left(x,t\right)$ [illegible]

$$\left\|\psi\left(x,t\right)\right\| \le C\left\|x\right\|^l, \quad \left(l > 1\right), \tag{5}$$

[illegible] $\left\|x\right\|$ [illegible]

$$\left\|x\right\|_1 = \max_i \left|x_i\right|; \quad \left\|x\right\|_2 = \sum_{i=1}^{n}\left|x_i\right|; \quad \left\|x\right\|_3 = \sqrt{\sum_{i=1}^{n}\left|x_i\right|^2}.$$

[illegible] (1) k-[illegible]

$$\sum_{s=0}^{k}\left\{\alpha_s x_{m+s}+h\beta_s \dot{x}_{m+s}\right\}=0 . \qquad (6)$$

[illegible]

$$\sum_{s=0}^{k}\left\{\alpha_s x_{m+s}+h\beta_s f\left(x_{m+s},t_{m+s}\right)\right\}=0 , \qquad (7)$$

[illegible] k -[illegible] (6) [illegible] (1) [illegible] (7) [illegible] $x(t)$ [illegible] (1) [illegible] h [illegible] $f(x,t)$. [illegible] [2] [illegible]

$$L\left[x(t)\right]=\sum_{s=0}^{k}\left\{\alpha_s x\left(t+s_h\right)+h\beta_s \dot{x}\left(t+s_h\right)\right\}$$

[illegible] h [illegible] $x(t)$. [illegible] α_s,β_s [illegible] (7) [illegible] (6).

[illegible] $L\left[x(t)\right]=O\left(h^{p+1}\right)$ [illegible] $x(t)$ [illegible] $p+1$ [illegible]

$$\sum_{s=0}^{k}\alpha_s=0 , \qquad (8)$$

$$\sum_{s=0}^{k}\left\{\alpha_s\frac{s^{\nu}}{\nu!}+\beta_s\frac{s^{\nu}}{(\nu-1)!}\right\}=0 ,\ [illegible]\ \nu=1,2,\ldots,p . \qquad (9)$$

[illegible] (6) [illegible] α_s [illegible] β_s $\left(s=1,2,,k\right)$ [illegible] (8) [illegible] (9).

[illegible] (6) $\alpha_k\neq 0$, $\beta_k=0$ - [illegible] k -[illegible] (4) [illegible]

$$\sum_{s=0}^{k}\left\{\alpha_s E+h\beta_s A_{m+s}\right\}x_{m+s}=0 , \qquad (10)$$

[illegible] E - [illegible] $n\times n$, $A_s=A\left(t+hs\right)$. [illegible] $\alpha_k\neq 0$, [illegible] (10) [illegible] α_k, [illegible]

$$\sum_{s=0}^{k}\left\{a_s E+hb_s A_{m+s}\right\}x_{m+s}=0 ,$$

[illegible] $a_k=1,\ b_k=0,\ a_s=\dfrac{\alpha_s}{\alpha_k},\ b_s=\dfrac{\beta_s}{\beta_k},\quad \left(s=0,1,\ldots,k-1\right)$. [illegible] $\left(n\cdot k\right)$ - [illegible]

$$z_m=\left(x_{m+k},x_{m+k-1},\ldots,x_{m+1}\right)^T .$$

[illegible] (10) [illegible]

$$z_m=B_{m-1}z_{m-1} , \qquad (11)$$

[illegible] B_{m-1} [illegible] $\left(n\cdot k\right)\times\left(n\cdot k\right)$ - [illegible]

$$B_{m-1}=\begin{pmatrix} D_{k-1} & D_{k-2} & \ldots & D_1 & D_0\\ E & \theta & \ldots & \theta & \theta\\ \theta & E & \ldots & \theta & \theta\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ \theta & \theta & \ldots & E & \theta \end{pmatrix} \qquad (12)$$

[illegible] D_i – n -[illegible]

$$D_i = -\left(a_i E + h b_i A_{m+i}\right), \quad \left(i = k-1, \ldots, 0\right).$$

[illegible] (3)

$$z_m = B_{m-1} z_{m-1} + G g_{m-1}, \tag{13}$$

[illegible] g_{m-1} – $\left(n \cdot k\right)$-[illegible]

$g_{m-1} = colon\left(\psi\left(x_{m+n-1}, t_{m+n-1}\right), \ldots, \psi\left(x_m, t_m\right)\right)$, G – $\left(n \cdot k\right) \times \left(n \cdot k\right)$ - [illegible]

$$G = \begin{pmatrix} -hb_{k-1}E & -hb_{k-2}E & \ldots & -hb_1 E & -hb_0 E \\ \theta & \theta & \ldots & \theta & \theta \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \theta & \theta & \ldots & \theta & \theta \end{pmatrix}. \tag{14}$$

[illegible] 1. [illegible] (3) [illegible] h [illegible] (13) [illegible] (11) [illegible] (13) [illegible].

[illegible] $y_m\left(z_m, t_m\right)$ [illegible]

$$y_m\left(z_m, t_m\right) = B_{m-1} z_{m-1}. \tag{15}$$

[illegible] V_m [illegible] $h < h_{\max}$:

$$\|y\| \le V_m\left(y\right) \le L\|y\|,$$

$$V_m\left(y_m\right) - V_{m-1}\left(y_{m-1}\right) \le -\left(1-q\right) V_{m-1}\left(y_{m-1}\right), \tag{16}$$

[illegible] $\|\cdot\|$ - [illegible]

$$V_m\left(z_m\right) - V_{m-1}\left(z_{m-1}\right) = V_m\left(y_m\left(z_m, t_m\right)\right) - V_{m-1}\left(z_{m-1}\right) + V_m\left(z_m\right) - V_m\left(y_m\left(z_m, t_m\right)\right) \le$$

$$\le -\left(1-q\right) V_{m-1}\left(z_{m-1}\right) + L\left\|z_m - y_m\left(z_m, t_m\right)\right\|.$$

[illegible] $\left\|z_m - y_m\left(z_m, t_m\right)\right\|$. [illegible] (15) [illegible] (13) [illegible]

$$\left\|z_m - y_m\left(z_m, t_m\right)\right\| = \left\|G g_{m-1}\right\| \le \|G\| \cdot \left\|g_{m-1}\right\| \le h \sum_{j=1}^{k} \left|b_{k-j}\right| C_1 \left\|z_{m-1}\right\|^l \le h\gamma_1 \left\|z_{m-1}\right\| \le h\gamma_1 V_{m-1}\left(z_{m-1}\right),$$

[illegible] $\gamma_1 = C_1 \sum_{j=1}^{k} \left|b_{k-j}\right| \cdot \left\|z_{m-1}\right\|^{l-1}$, [illegible] C_1 - [illegible]

$$V_m\left(z_m\right) - V_{m-1}\left(z_{m-1}\right) \le -\left(1-q\right) V_{m-1}\left(z_{m-1}\right) + hL\gamma_1 V_{m-1}\left(z_{m-1}\right).$$

[illegible] $V_m\left(z_m\right) - V_{m-1}\left(z_{m-1}\right) \le -\left(1 - q - hL\gamma_1\right) V_{m-1}\left(z_{m-1}\right)$.

[illegible] (13) [illegible]

$$1 - q - hL\gamma_1 > 0, \quad h < h_{\max} = \frac{1-q}{L\gamma_1}. \tag{17}$$

[illegible] $\psi\left(x, t\right)$ [illegible] (5), [illegible] δ -[illegible] $\|x\| < \delta$ [illegible] (17) [illegible] [3] [illegible] (13) [illegible].

[illegible] (3) k -[illegible] (6). [illegible] (6) [illegible] α_s [illegible] β_s $\left(s = 1, \ldots, k\right)$ [illegible] (8), (9) [illegible] $\alpha_k \ne 0$ [illegible] $\beta_k \ne 0$ - [illegible] k -[illegible]. [illegible] (6) [illegible] (4) [illegible] (10). [illegible]

$$\alpha_k E + h\beta_k A_{m+k}, \quad m \in [0;1;\dots)$$

[illegible] (10) [illegible]

$$x_{m+k} = -\sum_{s=0}^{k-1} \left(\alpha_k E + h\beta_k A_{m+k}\right)^{-1} \left(\alpha_s E + h\beta_s A_{m+s}\right) x_{m+s}. \tag{18}$$

[illegible] (18) [illegible]

$$z_m = B_{m-1}^{(1)} z_{m-1}, \tag{19}$$

[illegible] z_m [illegible] $(n \cdot k)$-[illegible], $B_{m-1}^{(1)}$ [illegible] $(n\cdot k)\times(n\cdot k)$-[illegible]

$$B_{m-1}^{(1)} = \begin{pmatrix} C_{k-1} & C_{k-2} & \dots & C_1 & C_0 \\ E & \theta & \dots & \theta & \theta \\ \theta & E & \dots & \theta & \theta \\ \dots & \dots & \dots & \dots & \dots \\ \theta & \dots & \dots & E & \theta \end{pmatrix} \tag{20}$$

[illegible] C_i [illegible] n-[illegible]

$$C_i = -\left(\alpha E + h\beta_k A_{m+k}\right)^{-1} \left(\alpha_i E + h\beta_i A_{m+i}\right) \quad (i = k-1, \dots, 0).$$

[illegible] (3)

$$z_m = B_{m-1}^{(1)} z_{m-1} + G_{m-1}^{(1)} g_{m-1} + G_m^{(2)} g_m, \tag{21}$$

[illegible] $G_{m-1}^{(1)}$ [illegible] $G_m^{(2)}$ [illegible] $(n\cdot k)\times(n\cdot k)$-[illegible]:

$$G_{m-1}^{(1)} = \begin{pmatrix} -h\beta_{k-1}\left(\alpha_k E + h\beta_k A_{m+1}\right)^{-1} & \dots & -h\beta_0\left(\alpha_k E + h\beta_k A_{m+k}\right)^{-1} \\ \theta & \dots & \theta \\ \dots & \dots & \dots \\ \theta & \dots & \theta \end{pmatrix},$$

$$G_m^{(2)} = \begin{pmatrix} -h\beta_k\left(\alpha_k E + h\beta_k A_{m+k}\right)^{-1} & \theta & \dots & \theta \\ \theta & \theta & \dots & \theta \\ \dots & \dots & \dots & \dots \\ \theta & \theta & \dots & \theta \end{pmatrix}.$$

[illegible] (21) [illegible] [4]

$$\det\left(G_m^{(2)} \left(\frac{\partial g}{\partial z}\right)_{|Z=Z_m} - E\right) \neq 0 \quad [illegible] \quad \forall z \in R^{n\cdot k}$$

$$\left\| G_m^{(2)} g_m(z_m, t_m) - z_m \right\| \to \infty, \quad [illegible] \quad \|z\| \to \infty. \tag{22}$$

[illegible] 2. [illegible] (3) [illegible] (21) [illegible] (22) [illegible] h [illegible] (21) [illegible] (19) [illegible] (21) [illegible] $h < h_{\max}$.

[illegible] $y_m(z_m, t_m)$ [illegible]

$$y_m(z_m, t_m) = B_{m-1}^{(1)} z_{m-1}. \tag{23}$$

[illegible] V_m, [illegible] (16). [illegible] 1 [illegible] $V_m(z_m) - V_{m-1}(z_{m-1})$

$$V_m(z_m) - V_{m-1}(z_{m-1}) \le -(1-q) V_{m-1}(z_{m-1}) + L \left\| z_m - y_m(z_m, t_m) \right\|.$$

[illegible] $\left\| z_m - y_m(z_m, t_m) \right\|$. [illegible] (23) [illegible] (21) [illegible]

$$\left\|z_m - y_m\left(z_m, t_m\right)\right\| = \left\|G_{m-1}^{(1)} g_{m-1} + G_m^{(2)} g_m\right\| \le \left\|G_{m-1}^{(1)}\right\| \cdot \left\|g_{m-1}\right\| + \left\|G_m^{(2)}\right\| \cdot \left\|g_m\right\| \le$$

$$\le \sum_{j=1}^{k} \frac{\left|\beta_{k-j}\right|}{\left|\beta_j\right| \cdot \left\|A_{m+k}\right\|} \cdot \left\|g_{m-1}\right\| + \frac{\left\|g_m\right\|}{\left\|A_{m+k}\right\|} \le \xi_1\left(z_{m-1}\right)\left\|z_{m-1}\right\| + \xi_2\left(z_m\right)\left\|z_m\right\| \le$$

$$\le \xi_1\left(z_{m-1}\right) V_{m-1}\left(z_{m-1}\right) + \xi_2\left(z_m\right) V_m\left(z_m\right),$$

[illegible] $\xi_1\left(z_{m+1}\right) = \sum_{j=1}^{k} \frac{\left|\beta_{k-j}\right| \cdot C_1}{\left|\beta_j\right| \cdot \left\|A_{m+k}\right\|} \cdot \left\|z_{m-1}\right\|^{l-1}; \quad \xi_2\left(z_m\right) = \frac{C_2}{\left\|A_{m+k}\right\|} \cdot \left\|z_m\right\|^{l-1},$ [illegible] C_1 [illegible] C_2 - [illegible]

[illegible]

$$V_m\left(z_m\right) - V_{m-1}\left(z_{m-1}\right) \le -(1-q) V_{m-1}\left(z_{m-1}\right) + L\xi_1 V_{m-1}\left(z_{m-1}\right) + L\xi_2 V_m\left(z_m\right).$$

[illegible] $$V_m\left(z_m\right) - V_{m-1}\left(z_{m-1}\right) \le \frac{-\left(1 - q - L\left(\xi_1 + \xi_2\right)\right)}{1 - L\xi_2} V_{m-1}\left(z_{m-1}\right).$$

[illegible] (21) [illegible]

$$1 - q - L\left(\xi_1 + \xi_2\right) > 0, \ 1 - L\xi_1 > 0.$$

[illegible] $$\xi_1 + \xi_2 < \frac{1-q}{L}. \tag{24}$$

[illegible] $\psi(x,t)$ [illegible] (5), [illegible] $\|x\| < \delta$ [illegible] $h < h_{\max}$, [illegible] (23) [illegible] (24) [illegible] [3] [illegible] (21) [illegible].

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