

519.622

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... .. k - , .. k - . () : , , k - ,

$$\frac{dx}{dt} = f(x, t), \tag{1}$$

$$f_i(x, t) \in C_{x,t}^{(1,0)}(D), \tag{2}$$

$$D = \{0 < t < \infty; x \in \square^n\}.$$

$$(2) \quad \dots \quad f(x, t) \quad D \quad \|x\| < \delta$$

$$f(x, t) = A(t)x + \psi(x, t),$$

$$A(t) = \left(\frac{\partial f(x, t)}{\partial x} \right)_{|x=0}, \quad \psi(x, t) \quad \|x\| < \delta \tag{1}$$

$$\frac{dx}{dt} = A(t)x + \psi(x, t). \tag{3}$$

$$[1] \quad \dots \quad \psi(x, t) \quad x=0$$

$$\frac{dx}{dt} = A(t) \cdot x \tag{4}$$

$$\dots \quad (3), \quad (1) \quad \psi(x, t)$$

$$\|\psi(x, t)\| \leq C \|x\|^l, \quad (l > 1), \tag{5}$$

$\|x\|$

$$\|x\|_1 = \max_i |x_i|; \quad \|x\|_2 = \sum_{i=1}^n |x_i|; \quad \|x\|_3 = \sqrt{\sum_{i=1}^n |x_i|^2}.$$

... .. (1) k -

$$\sum_{s=0}^k \{\alpha_s x_{m+s} + h\beta_s \dot{x}_{m+s}\} = 0. \quad (6)$$

$$\sum_{s=0}^k \{\alpha_s x_{m+s} + h\beta_s f(x_{m+s}, t_{m+s})\} = 0, \quad (7)$$

$$k - \quad (6) \quad (1)$$

$$, \quad (7)$$

$$\emptyset \quad x(t) \quad (1) \quad h$$

$$f(x, t). \quad [2]$$

$$L[x(t)] = \sum_{s=0}^k \{\alpha_s x(t+s_h) + h\beta_s \dot{x}(t+s_h)\}$$

$$h \quad x(t).$$

$$\alpha_s, \beta_s \quad (7) \quad (6).$$

$$L[x(t)] = O(h^{p+1}) \quad x(t),$$

$p+1$

$$\sum_{s=0}^k \alpha_s = 0, \quad (8)$$

$$\sum_{s=0}^k \left\{ \alpha_s \frac{s^\nu}{\nu!} + \beta_s \frac{s^\nu}{(\nu-1)!} \right\} = 0, \quad \nu = 1, 2, \dots, p. \quad (9)$$

$$, \quad (6) \quad \alpha_s \quad \beta_s \quad (s = 1, 2, \dots, k)$$

(8) (9).

$$(6) \quad \alpha_k \neq 0, \quad \beta_k = 0 \quad k - \quad (4)$$

$$\sum_{s=0}^k \{\alpha_s E + h\beta_s A_{m+s}\} x_{m+s} = 0, \quad (10)$$

$$E - \quad n \times n, \quad A_s = A(t+hs). \quad \alpha_k \neq 0,$$

(10) $\alpha_k,$

$$\sum_{s=0}^k \{a_s E + hb_s A_{m+s}\} x_{m+s} = 0,$$

$$a_k = 1, \quad b_k = 0, \quad a_s = \frac{\alpha_s}{\alpha_k}, \quad b_s = \frac{\beta_s}{\beta_k}, \quad (s = 0, 1, \dots, k-1). \quad (n \cdot k) -$$

$$z_m = (x_{m+k}, x_{m+k-1}, \dots, x_{m+1})^T. \quad (10)$$

$$z_m = B_{m-1} z_{m-1}, \quad (11)$$

B_{m-1} ó $(n \cdot k) \times (n \cdot k)$ -

$$B_{m-1} = \begin{pmatrix} D_{k-1} & D_{k-2} & \dots & D_1 & D_0 \\ E & \theta & \dots & \theta & \theta \\ \theta & E & \dots & \theta & \theta \\ \dots & \dots & \dots & \dots & \dots \\ \theta & \theta & \dots & E & \theta \end{pmatrix} \quad (12)$$

D_i ó n -

$$D_i = -(a_i E + h b_i A_{m+i}), \quad (i = k-1, \dots, 0).$$

(3)

$$z_m = B_{m-1} z_{m-1} + G g_{m-1}, \quad (13)$$

g_{m-1} ó $(n \cdot k)$ -

$g_{m-1} = \text{colon}(\psi(x_{m+n-1}, t_{m+n-1}), \dots, \psi(x_m, t_m))$, G ó $(n \cdot k) \times (n \cdot k)$ -

$$G = \begin{pmatrix} -hb_{k-1}E & -hb_{k-2}E & \dots & -hb_1E & -hb_0E \\ \theta & \theta & \dots & \theta & \theta \\ \dots & \dots & \dots & \dots & \dots \\ \theta & \theta & \dots & \theta & \theta \end{pmatrix}. \quad (14)$$

h I .

(11)

(13)

(3)

 \emptyset \emptyset

(13)

$y_m(z_m, t_m)$ \emptyset ,

$$y_m(z_m, t_m) = B_{m-1} z_{m-1}. \quad (15)$$

 V_m

$h < h_{\max}$:

$$\|y\| \leq V_m(y) \leq L\|y\|,$$

$$V_m(y_m) - V_{m-1}(y_{m-1}) \leq -(1-q)V_{m-1}(y_{m-1}), \quad (16)$$

$\|\cdot\|$ -

$$\begin{aligned} V_m(z_m) - V_{m-1}(z_{m-1}) &= V_m(y_m(z_m, t_m)) - V_{m-1}(z_{m-1}) + V_m(z_m) - V_m(y_m(z_m, t_m)) \\ &\leq -(1-q)V_{m-1}(z_{m-1}) + L\|z_m - y_m(z_m, t_m)\|. \end{aligned}$$

$$\|z_m - y_m(z_m, t_m)\|. \quad (15) \quad (13)$$

$$\|z_m - y_m(z_m, t_m)\| = \|G g_{m-1}\| \leq \|G\| \cdot \|g_{m-1}\| \leq h \sum_{j=1}^k |b_{k-j}| C_1 \|z_{m-1}\|^l \leq h \gamma_1 \|z_{m-1}\| \leq h \gamma_1 V_{m-1}(z_{m-1}),$$

$$\gamma_1 = C_1 \sum_{j=1}^k |b_{k-j}| \cdot \|z_{m-1}\|^{l-1}, \quad C_1 -$$

$$V_m(z_m) - V_{m-1}(z_{m-1}) \leq -(1-q)V_{m-1}(z_{m-1}) + hL\gamma_1 V_{m-1}(z_{m-1}).$$

$$V_m(z_m) - V_{m-1}(z_{m-1}) \leq -(1-q - hL\gamma_1)V_{m-1}(z_{m-1}).$$

\emptyset (13)

$$1 - q - hL\gamma_1 > 0, \quad h < h_{\max} = \frac{1-q}{L\gamma_1}. \quad (17)$$

$\psi(x, t)$

(5)

 δ -

$\|x\| < \delta$,

(17)

[3]

 \emptyset

(13)

(3) k -

(6).

(6)

 $\alpha_s \quad \beta_s \quad (s = 1, \dots, k)$

(8), (9),

 $\alpha_k \neq 0$ $\beta_k \neq 0$ k -

(6)

(4)

(10).

$$\alpha_k E + h\beta_k A_{m+k}, \quad m \in [0; 1; \dots]$$

(10)

$$x_{m+k} = -\sum_{s=0}^{k-1} (\alpha_k E + h\beta_k A_{m+k})^{-1} (\alpha_s E + h\beta_s A_{m+s}) x_{m+s}. \quad (18)$$

(18)

$$z_m = B_{m-1}^{(1)} z_{m-1}, \quad (19)$$

 z_m ó $(n \cdot k)$ - , $B_{m-1}^{(1)}$ ó $(n \cdot k) \times (n \cdot k)$ -

$$B_{m-1}^{(1)} = \begin{pmatrix} C_{k-1} & C_{k-2} & \dots & C_1 & C_0 \\ E & \theta & \dots & \theta & \theta \\ \theta & E & \dots & \theta & \theta \\ \dots & \dots & \dots & \dots & \dots \\ \theta & \dots & \dots & E & \theta \end{pmatrix} \quad (20)$$

 C_i ó n -

$$C_i = -(\alpha E + h\beta_k A_{m+k})^{-1} (\alpha_i E + h\beta_i A_{m+i}) \quad (i = k-1, \dots, 0).$$

(3)

$$z_m = B_{m-1}^{(1)} z_{m-1} + G_{m-1}^{(1)} g_{m-1} + G_m^{(2)} g_m, \quad (21)$$

 $G_{m-1}^{(1)}$ $G_m^{(2)}$ ó $(n \cdot k) \times (n \cdot k)$ -

$$G_{m-1}^{(1)} = \begin{pmatrix} -h\beta_{k-1} (\alpha_k E + h\beta_k A_{m+k})^{-1} & \dots & -h\beta_0 (\alpha_k E + h\beta_k A_{m+k})^{-1} \\ \theta & \dots & \theta \\ \dots & \dots & \dots \\ \theta & \dots & \theta \end{pmatrix},$$

$$G_m^{(2)} = \begin{pmatrix} -h\beta_k (\alpha_k E + h\beta_k A_{m+k})^{-1} & \theta & \dots & \theta \\ \theta & \theta & \dots & \theta \\ \dots & \dots & \dots & \dots \\ \theta & \theta & \dots & \theta \end{pmatrix}.$$

(21)

[4]

$$\det \left(G_m^{(2)} \left(\frac{\partial g}{\partial z} \right)_{z=z_m} - E \right) \neq 0 \quad \forall z \in R^{n \cdot k}$$

$$\|G_m^{(2)} g_m(z_m, t_m) - z_m\| \rightarrow \infty, \quad \|z\| \rightarrow \infty. \quad (22)$$

(21) 2. \emptyset (22) h (21) ,

\emptyset (21) $h < h_{\max}$.

$$y_m(z_m, t_m) = B_{m-1}^{(1)} z_{m-1}. \quad (23)$$

 V_m ,

(16). , 1

 $V_m(z_m) - V_{m-1}(z_{m-1})$

$$V_m(z_m) - V_{m-1}(z_{m-1}) \leq -(1-q)V_{m-1}(z_{m-1}) + L \|z_m - y_m(z_m, t_m)\|.$$

$$\|z_m - y_m(z_m, t_m)\|. \quad (23) \quad (21)$$

$$\begin{aligned} \|z_m - y_m(z_m, t_m)\| &= \|G_{m-1}^{(1)}g_{m-1} + G_m^{(2)}g_m\| \leq \|G_{m-1}^{(1)}\| \cdot \|g_{m-1}\| + \|G_m^{(2)}\| \cdot \|g_m\| \leq \\ &\leq \sum_{j=1}^k \frac{|\beta_{k-j}|}{|\beta_j| \cdot \|A_{m+k}\|} \cdot \|g_{m-1}\| + \frac{\|g_m\|}{\|A_{m+k}\|} \leq \xi_1(z_{m-1})\|z_{m-1}\| + \xi_2(z_m)\|z_m\| \leq \\ &\leq \xi_1(z_{m-1})V_{m-1}(z_{m-1}) + \xi_2(z_m)V_m(z_m), \\ \xi_1(z_{m+1}) &= \sum_{j=1}^k \frac{|\beta_{k-j}| \cdot C_1}{|\beta_j| \cdot \|A_{m+k}\|} \cdot \|z_{m-1}\|^{l-1}; \quad \xi_2(z_m) = \frac{C_2}{\|A_{m+k}\|} \cdot \|z_m\|^{l-1}, \quad C_1 \quad C_2 \end{aligned}$$

$$V_m(z_m) - V_{m-1}(z_{m-1}) \leq -(1-q)V_{m-1}(z_{m-1}) + L\xi_1 V_{m-1}(z_{m-1}) + L\xi_2 V_m(z_m).$$

$$V_m(z_m) - V_{m-1}(z_{m-1}) \leq \frac{-(1-q-L(\xi_1 + \xi_2))}{1-L\xi_2} V_{m-1}(z_{m-1}).$$

$$(21) \quad 1 - q - L(\xi_1 + \xi_2) > 0, \quad 1 - L\xi_1 > 0.$$

$$\xi_1 + \xi_2 < \frac{1-q}{L}. \tag{24}$$

$$\psi(x, t) \tag{5}$$

$$\|x\| < \delta \tag{23}$$

$$h < h_{\max},$$

$$[3] \tag{24}$$

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