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1. $f(z)$ ó $\{z_n\}$, $f(0) = 1$

$$M(r, f) = \max_{0 \leq \theta \leq 2\pi} |f(re^{i\theta})|, \quad L(r, f) = \min_{0 \leq \theta \leq 2\pi} |f(re^{i\theta})|,$$

$$n(r) = n(r, 0, f) = \sum_{|z_n| \leq r} 1, \quad N(r) = \int_0^r \frac{n(t)}{t} dt.$$

, ρ f

$$\rho = \lim_{r \rightarrow +\infty} \frac{\overline{\ln \ln M(r, f)}}{\ln r}.$$

ε_ρ ó ρ ,

$$c(f) = \lim_{r \rightarrow +\infty} \frac{n(r)}{\ln M(r, f)}.$$

$$C(\rho) = \inf_{f \in \varepsilon_\rho} c(f).$$

[1] [2,3] ,

$$C(\rho) = \frac{1}{\pi} \sin \pi \rho, \quad 0 \leq \rho \leq 1; \tag{1}$$

$$\frac{|\sin \pi \theta|}{A_0(1 + \ln r)|\sin \pi \rho| + \pi} \leq C(\rho) \leq \frac{1}{\pi} |\sin \pi \rho|, \quad \rho > 1, \tag{2}$$

A_0 ó

(2)

$$f_\rho(z) = \prod_{n=1}^{+\infty} \left(1 + \frac{z}{n^\sigma} \right) \exp \left(\sum_{k=1}^q \frac{1}{k} \left(\frac{z}{-n^\sigma} \right)^k \right),$$

$$\sigma = \rho^{-1}, \quad q = [\rho], \quad \rho \notin \mathbb{N}, \quad n(r) \sim r^\rho \text{ i } \ln M(r, f) \sim \frac{\pi}{|\sin \pi \rho|} r^\rho \quad r \rightarrow +\infty.$$

[4] f $\rho > 1,$

$$C(\rho) < \frac{|\sin \pi \rho|}{\pi}.$$

$$\frac{C(\rho)}{C(\rho)} \quad \rho > 1 \quad \rho \rightarrow +\infty.$$

1 < ρ < + ∞

[5]

$$c(f) \geq A |\sin \pi \rho| \tag{3}$$

$$f \quad \rho, \quad \{z_n\} \quad \arg z = \pi.$$

[5]

(z) \emptyset

$$\ln(z) = (-1)^q z^{q+1} \int_0^{+\infty} \frac{n(t, 0)}{t^{q+1}(t+z)} dt,$$

$$q = [\rho], \quad |\arg z| < \pi.$$

$$B(r, \theta) = \sup_{|\theta| < \pi} |\ln(re^{i\theta})|,$$

$$\varphi(r) = \frac{B(r, \theta)}{r^{q+1}}, \quad \varphi(r) = \frac{n(r, 0)}{r^{q+1}}.$$

$$\varphi(r) \quad r \leq 0$$

$$\varphi(r) \leq 12M\varphi(r) + \pi H^* \varphi(r) + 10 \int_0^{+\infty} \frac{\varphi(t)}{t+r} dt, \tag{4}$$

$$M\varphi(r) = \sup_{\varepsilon > 0} \frac{1}{2\varepsilon} \int_{|t-r| < \varepsilon} \varphi(t) dt$$

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$$H^* \varphi(r) = \frac{1}{\pi} \sup_{\varepsilon > 0} \left| \int_{|t-r| > \varepsilon} \frac{\varphi(t) dt}{t-r} \right|$$

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(4) [5] :

$$\left(\int_{R_n}^{+\infty} \left(\frac{B(r, f)}{r^{q+1}} \right)^p dr \right)^{1/p} \leq \frac{1}{A \sin \frac{\pi}{p}} \left(\int_{R_n}^{+\infty} \left(\frac{n(r, 0, f)}{r^{q+1}} \right)^p dr \right)^{1/p}, \tag{5}$$

$$p = \frac{1}{q+1-\rho} + \varepsilon_n, \quad R_n \rightarrow +\infty, \quad \varepsilon_n \rightarrow 0 \quad n \rightarrow +\infty \quad \acute{o} . \tag{3}$$

$$(5).$$

[6] - (4), 4 .

$$(r) \leq \pi M \varphi(r) + \pi M (H\varphi)r + \int_0^{+\infty} \frac{\varphi(t)}{t+r} dt, \tag{6}$$

$$H\varphi(r) = \frac{1}{\pi} \lim_{\varepsilon \rightarrow 0} \int_{|t-r|>\varepsilon} \frac{\varphi(t)}{r-t} dt$$

ó .

(6) :

$$(r) \leq \pi \varphi^\perp(r) + \pi \tilde{\varphi}^\perp(r) + \int_0^{+\infty} \frac{\varphi(t)}{t+r} dt, \tag{7}$$

$$\varphi^\perp(r) = \sup_{y>0} \left| \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{y\varphi(t)dt}{(r-t)^2 + y^2} \right|$$

ó ,

$$\tilde{\varphi}^\perp(r) = \sup_{y>0} \left| \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{(r-t)\varphi(t)dt}{(r-t)^2 + y^2} \right|$$

ó .

$\|M\varphi\|_p$ $\|M(H\varphi)\|_p$ $M\varphi$ $M(H\varphi)$ L_p ó $\|\varphi_\alpha^*\|_p$ $\|\tilde{\varphi}_\alpha^*\|_p$

$$\varphi_\alpha^*(r) = \sup_{\alpha(r)} \left| \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{y\varphi(t)dt}{(x-t)^2 + y^2} \right|$$

$$\tilde{\Phi}_\alpha^*(r) = \sup_{\alpha(r)} \left| \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{(x-t)\varphi(t)dt}{(x-t)^2 + y^2} \right|$$

$$\alpha(r) = \{x + iy : |x - r| < \alpha y, \alpha > 0, y > 0\},$$

([7, .239])

$$\|\tilde{\varphi}_\alpha^*\|_p \leq C_p \|\varphi_\alpha^*\|_p, \quad p > 1,$$

$$C_p = \operatorname{tg} \frac{\pi}{2p} \quad 1 < p \leq 2 \quad C_p = \operatorname{ctg} \frac{\pi}{2p} \quad \geq 2.$$

(3).

$$(3) \quad A = \frac{1}{4\pi},$$

$$\frac{|\sin \pi \rho|}{4\pi} \leq c(f) \leq \frac{|\sin \pi \rho|}{\pi}$$

$f \in \mathcal{E}_\rho$

[5],

f $\rho > 1$,

k

2.

1. $f \in \mathcal{E}_\rho$ $\rho > 1$ \emptyset , $f(0) = 1$,

$$B(r, f) = \sup_{|\theta| < \pi} |\ln f(re)^{i\theta}|.$$

$$\overline{\lim}_{r \rightarrow +\infty} \frac{n(r)}{B(r, f)} \geq \frac{|\sin \pi \rho|}{4\pi}.$$

1. $f \in \mathcal{E}_\rho$ $\rho > 1$ \emptyset , $f(0) = 1$.

$$\overline{\lim}_{r \rightarrow +\infty} \frac{N(r)}{B(r, f)} \geq \frac{\sin \pi \rho}{4\pi \rho}.$$

2. $f \in \mathcal{E}_\rho$ $\rho > 1, f(0) = 1$ f

k

$$\overline{\lim}_{r \rightarrow +\infty} \frac{n(r)}{\ln M(r, f)} \geq \frac{|\sin \pi \rho|}{4\pi k}, \quad \overline{\lim}_{r \rightarrow +\infty} \frac{\ln L(r, f)}{n(r)} \geq -\frac{4\pi k}{|\sin \pi \rho|},$$

$$\overline{\lim}_{r \rightarrow +\infty} \frac{N(r)}{\ln M(r, f)} \geq \frac{|\sin \pi \rho|}{4\pi \rho k}, \quad \overline{\lim}_{r \rightarrow +\infty} \frac{\ln L(r, f)}{N(r)} \geq -\frac{4\pi \rho k}{|\sin \pi \rho|}.$$

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