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MATHEMATICAL MODELING OF STRESSES AND STRAINS OF WOOD IN THE DRYING PROCESS

Introduction. For rational directing of wood drying process provided with necessary quality indexes is an important problem to make physical-mathematical model of rheological state of wood taking into account mechanism of regenerating strains in time, and also to develop methods of synthesis and analysis of strain relaxational processes in these materials taking into account changeable potentials of mass heat transfer, interrelation between stresses and parameters of external and internal mass heat transfer, anisotropy of properties, structure transformations.

Thermodynamical approach and mathematical model. In base of synthesis and analysis of offered physical-mathematical model of strains and stresses development in technological wood drying processes the methodology of system-structural approach, that stipulated basing of parameters choice of phenomenological model description and setting of suitable heat, mass, energy and impulse laws, is laid based on of thermodynamically analysis structure of relations was described mutual influence of different phenomena and factors specifically creeping processes, relaxation and shrinkage on deformation of arboreal materials was set. There was described generalized function of free energy (thermodynamical potential of researching system), as material state function with changeable potentials of mass heat transfer, that is determined by components of summary (elastic, viscous-elastic and residual) strains tensors or stresses, chemical potential or moisture content and temperature T, and entropys. State function is expressed through invariants from determining parameters, specifically strains tensors.

There was a synthesized physical-mathematical model of strain-relaxational processes in wood durying process in general case directed in the form of differential equations system : balance and motion equations

$$\sigma_{ij,i} = \rho \partial^2 U_i / \partial \tau^2; \quad \sigma_{ij} = \sigma_{ji}; \quad (1)$$

$$\frac{\partial t}{\partial \tau} = \text{div}(\lambda_{tt} \text{grad} t) + \text{div}(\lambda_{tu} \text{grad} U) + \text{div}(\lambda_{tp} \text{grad} P) + t \frac{\partial}{\partial \tau} \left(\varepsilon_{kk} \frac{\partial G(t, U, P)}{\partial t} \right); \quad (2)$$

$$\frac{\partial U}{\partial \tau} = \text{div}(\lambda_{ut} \text{grad} t) + \text{div}(\lambda_{uu} \text{grad} U) + \text{div}(\lambda_{up} \text{grad} P) + U \frac{\partial}{\partial \tau} \left(\varepsilon_{kk} \frac{\partial G(t, U, P)}{\partial U} \right); \quad (3)$$

$$\frac{\partial P}{\partial \tau} = \text{div}(\lambda_{pt} \text{grad} t) + \text{div}(\lambda_{pu} \text{grad} U) + \text{div}(\lambda_{pp} \text{grad} P) + P_0 \frac{\partial \varepsilon_{kk}}{\partial \tau}; \quad (4)$$

$$G(t, U, P) = \frac{dV}{V_0} (3\lambda + 2\mu) - \xi P;$$

state equations:

$$\sigma_{ij} = 2\mu(1-\omega)\varepsilon_{ij} + \left(\varepsilon_{ij} \left(\lambda + \frac{2}{3}\mu\omega \right) - \frac{dV}{V_0} (3\lambda + 2\mu) + \xi P \right) \delta_{ij}; \quad (5)$$

$$\mu = \varepsilon \quad \gamma - \varepsilon_{ij} \frac{\partial}{\partial U} G(t, U, P) + \varepsilon_{ij} \varepsilon_{ij} \frac{\partial}{\partial U} \mu(1-\omega) + \varepsilon_{ij}^2 \frac{\partial}{\partial U} (\lambda/2 + \mu\omega/3); \quad (6)$$

$$S = \int_{t_0}^t \frac{C_v}{t} dt + \varepsilon_{ij} \frac{\partial}{\partial t} G(t, U, P) - \varepsilon_{ij} \varepsilon_{ij} \frac{\partial}{\partial T} \mu(1 - \omega) + \varepsilon_{ij}^2 \frac{\partial}{\partial T} (\lambda/2 - \mu \omega/3). \quad (7)$$

For enclosing of equation system (1)-(7) conditions of strains compatibility, cinematical equations of mass heat transfer potentials, and also initial and boundary conditions which are typical for different drying process stages were used.

The obtained equations system (1)-(7) allows to describe temperature-moisture and stress-strained fields in wood drying process with taking into account viscous-elastic properties of material and strains regeneration mechanism in time.

On basis of relation setting between balance equations (2)-(4) and generalized law of Hooke for viscous-elastic materials, the law of rheological wood behaviour with changeable temperature-moisture fields as viscous-elastic material, for which transformation in time of one kind of strains into another is characteristic was obtained. As long as for determination of characteristics of rheological wood behaviour in suitable change diapasons u and t were developed methods of experimental research on creeping and relaxation with use of integral functions (creeping and relaxation nucleus), rheological regularity of wood straining can be given in form.

$$\varepsilon(\tau, U, t) = \frac{1}{E(U, t)} \left(\sigma(\tau) + \int_0^\tau \left(\frac{\partial K_1(\tau - \tau_1 U_1 t)}{\partial \tau} + R_2(\tau - \tau_1 U_1 t) \frac{dU(\tau_1)}{d\tau_1} \right) \sigma(\tau_1) d\tau_1 \right) - \alpha U. \quad (8)$$

Offered rheological equation (8) was endorsed by experimental results for cases of creeping and reverse creeping strains for different levels of temperature and moisture.

Experimental researches of rheological wood properties.

Taking into account experimental researches referring to studies of stress-strained wood state in drying process, the following tasks of rheological researches were determined: to inquire into rheological behaviour of wood on creeping, relaxation and reverse creeping with taking into account anisotroption in range of change of moisture (8%, 15%, 20% Wrr) and temperature (20°C, 40°C, 80°C, 95°C); to set regularities in development of elastic, viscous-elastic and residual strains and to describe quantitatively function of creeping and relaxation, which are necessary for computation of stress-strained state of drying wood; to substantiate application of speed up methods of temporal analogies, which allow by results of short-time researches to forecast rheological properties of wood along fibres for long-time term. Each experiment was carried out on separately taken sample of pine-tree, fir-tree, birch and oak for constant loading tension and compression, that does not exceed limits of long-time resistance.

Results of experimental senses of creeping function $\psi(\tau, w, t=20^\circ\text{C})$ of pine-wood for tangential direction of deformation given in table 1, where $\psi(t=0) = E_n^{-1}$; $\psi(t=\infty) = E_t^{-1}$; E , E_t momentary and continuous modulus of elasticity, φ_1 characteristics allowing to estimate degree of wood creeping.

Table 1. Deformation indexes of pine-wood

$\Psi, 10^{-3}$ P		time of deformation $\tau \cdot 10^3$, ψ' -tension, ψ'' -compression									
		0.05	12	24	36	48	60	78	90	120	
ψ'	W=8%	1.07	1.21	1.33	1.43	1.50	1.54	1.58	1.63	1.68	
		ψ''	1.04	1.22	1.32	1.40	1.53	1.56	1.60	1.62	1.69
		φ	419	478	517	561	599	612	619	639	662
ψ'	W=15%	0.92	1.09	1.21	1.34	1.40	1.44	1.47	1.50	1.52	
		ψ''	0.97	1.07	1.26	1.36	1.42	1.44	1.46	1.50	1.54
		φ	346	378	432	483	503	510	520	533	545
ψ'	W=20%	0.86	1.05	1.18	1.27	1.36	1.37	1.40	1.42	1.43	
		ψ''	0.85	1.06	1.19	1.28	1.34	1.36	1.39	1.43	1.43
		φ	266	333	372	401	420	426	435	448	448
ψ'	W=W	0.64	0.82	0.93	0.99	1.03	1.06	1.06	1.07	1.07	
		ψ''	0.64	0.81	0.93	1.01	1.03	1.05	1.05	1.06	1.08
		φ	167	212	242	259	268	272	273	275	280

Results of strains researches of creeping and reverse creeping of wood across fibres allowed to build necessary for synthesis and analysis of stress-strained lumber state (8) of creeping functions with taking into account accumulation of residual strains:

$$K_1(\tau - \tau'; U, t) = \psi_0 + \sum_{k=1}^N a_k \exp(-\alpha_k (\tau - \tau')); \quad K_2(\tau - \tau'; U, t) = \beta \exp(-L(\tau - \tau'_1)). \quad (9)$$

Was developed an algorithm of determining parameters ψ_0 , a_k , α_k , β and substantiated a choice of their amount from minimum of quadratic deviation of approximation of experimental curves $\varepsilon(t, w, \tau)$. Application of more complicated function of creeping because of high changeability of strain properties of wood essentially complicates mathematical apparatus and it inconvenient for practice.

For research of rheological behaviour of wood along fibres and generalization of existing experimental data was offered an approach with use of fixed scale function, which provide invariance of strain process concerning temperature and moisture changes. Were built generalized curves of creeping for researched species of wood in half-logarithmical coordinates $(\varepsilon, \ln \tau)$ and determined scale function $\ln a(t, w)$, that characterizes creeping curves dependency on generalized, directed for base senses of temperature $t_0=40^\circ$ and moisture $W_0=8\%$. Was set that function is unlinear for each from arguments and is stipulated by their mutual influence. In general case $\ln a(t, w)$ is described by polynomial and coefficients are determined by method of minimum squares.

On base of observed peculiarities of wood straining along fibres for different temperature and moisture senses was offered rheological equation

$$\varepsilon(\tau) = \varepsilon_0(\sigma, t, W) + \frac{1}{n} \sum_{i=1}^n b_i (1 - \exp(-\tau a(t, W) / \tau_{oi})). \quad (10)$$

Were set spectrums of relaxation times and function $a(t, w)$. Specifically, for $t=40^\circ\text{C}$:

$$a(W) = (W_0/W)^b \frac{\ln 50(1 - (E_T(W_0)/E(W_0)))}{\ln 50(1 - (E_T(W)/E(W)))}; \quad \tau_{oi} = \tau_p(W_0)(W_0/W)^b \quad (11)$$

Senses of coefficient b for different species of wood and method of loading are directed in table 2.

Table 2. Senses of coefficients b for determining of relaxation time

		pine-tree	fir-tree	birch	oak
b	compression	0.63	0.51	0.42	0.7
	tension	4.73	4.3	3.85	4.8

Computation of flat stress-strained state of wood in drying process. With the use of elastic theory methods flat (two-dimensional) stress-strained wood state in drying process with taking into account anisotroption of mechanical and heat physical properties, stipulated by structural construction of wood was researched. An orthotropic plate of wood, which in two mutual perpendicular direction is characterized by anisotroption of mechanical and moisture properties, and a plane of orthotroption coincides with co-ordinates system planes, beginning of which is situated in the center of transversal section (fig.1).

Task of determining moisture stresses in drying wood by integration of equilibrium equations (1) turns into decision of differential equation with boundary conditions:

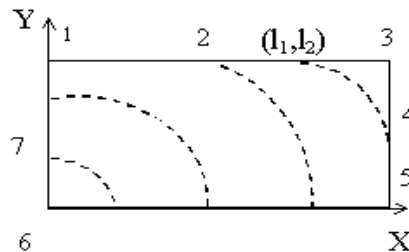


Fig. 1. Scheme of task organization of determining of stress-strained state of drying wood. l_1, l_2 – width and thickness of board.

$$\begin{aligned} \bar{\nabla} F &= -\partial^2(\beta_y \Delta U) / \partial x^2 - \partial^2(\beta_x \Delta U) / \partial y^2, \\ \bar{\nabla} &= E_y^1 \partial^4 / \partial x^4 + (G_{xy}^{-1} - 2\nu_{xy} / E_y) \partial^4 / \partial x^2 \partial y^2 + E_x \partial^4 / \partial y^4; \\ \sigma_x &= 0, X = \pm l_1 = L_1 / 2; \sigma_y = 0, Y = \pm l_2 = L_2 / 2; \\ \beta_y &= \left((\beta_{y0} - \beta_{x0}) \sqrt{(x^2 + y^2) / (l_1^2 + l_2^2)} + \beta_{x0} \right) (30 - W) / 30; \beta_x = \beta_{x0} (30 - W) / 30; \end{aligned} \quad (12)$$

$\beta_{x0}(W,t)$, $\beta_{y0}(W,t)$ ótangential and radial coefficients of shrinkage, $\Delta U = U^* - U(x,y,\tau)$ ó function of change of hygroscopic moisture content. For $U > U^*$ was taken $\Delta U = 0$, that accords with lack of shrinkage in moisture zone. For change of inflexibility in wood starts from 25% 30% then $U^* = 0.25$.

Function of stresses F is related with components of stresses by formulae

$$\sigma_x = \partial^2 F / \partial y^2; \sigma_y = \partial^2 F / \partial x^2; \tau_{xy} = -\partial^2 F / \partial x \partial y. \quad (13)$$

Numerical decision was obtained with the use of conjugate with distribution of temperature-moisture field designed function φ_1 that provide a minimum of potential energy of drying wood by dint of determination of unknown coefficients a and k from fixed system of equations

$$\sum_i^3 \sum_k^3 (\bar{\nabla} \varphi_e, \varphi_n)_{ik} = \left(-\partial^2(\beta_y \Delta U) / \partial x^2 - \partial^2(\beta_x \Delta U) / \partial y^2, \varphi_n \right), \quad (14)$$

where $(\Delta \varphi, \varphi)$ and right part (14) are scalar. Residual strains, stipulated by dependence between modules of elasticity E_x , E_y , ν_{xy} and moisture (effect of regeneration) are defined for formulas:

$$\sigma_{res} = \sigma - \sigma_{max} / 3; \sigma_{yres} = \sigma_y - \sigma_{ymax} / 3. \quad (15)$$

Numerical decision of written in criterional form of mass heat transfer equations (2),(3) and boundary conditions was obtained by intheratical locally one-dimensional method. On fig.2 and 3 are presented the modeling results of stress-strained state of drying wood.

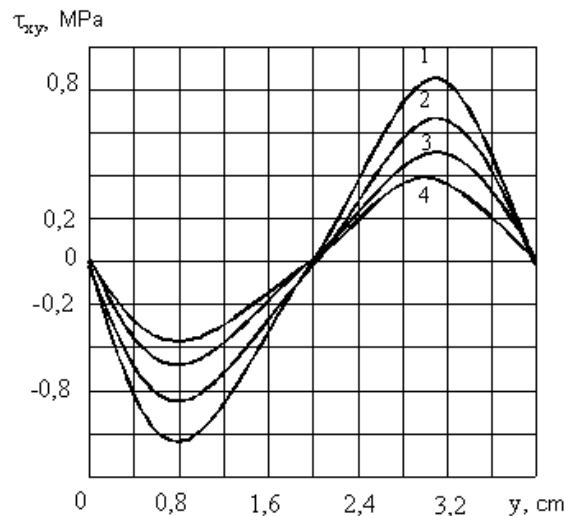


Fig. 2. Distribution of tangential stresses τ_{xy} ($\nu = 2$; $l_1 = 4$ cm; $l_2 = 8$ cm, curve 1 ó $F_0 \approx 2.47$; 2- $F_0 \approx 3.63$; 3 - $F_0 \approx 4.02$; 4 - $F_0 \approx 6.32$; $t_c = 84$ °C; $\varphi = 0.62$; $\alpha = 22.5$ W (m²t); $\beta = 3 \cdot 10^{-6}$ m/ ; $\beta_0 = 7.1$ %; $\beta_r = 3.4$ %; $\rho_0 = 460$ g/m³)

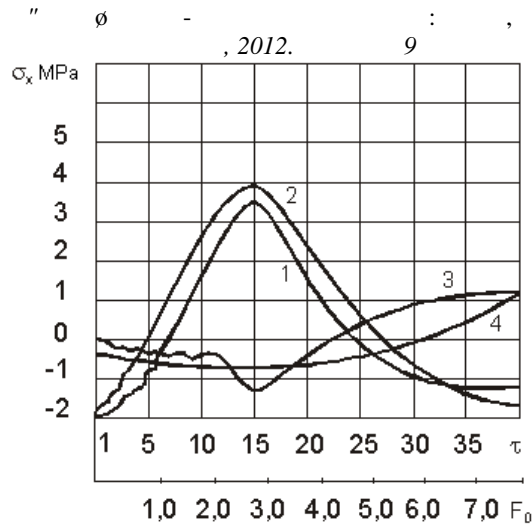


Fig. 3. Change of normal stresses σ_x (1 – point 1; 2 – point 2; 3 – point 6; 4 – point 7; $t_c=84$ °C; $\varphi=0,62$; $\alpha=22,5$ Wτ(m²t); $\beta=3 \cdot 10^{-6}$ m/c; $\beta_{y0}=7,1$ %; $\beta_{x0}=3,01$ %; $\rho_0=460$ kg/m³)

On basis of offered single approach was solved practically important for wood drying process class of task of stress-strained state determining. Was set a distribution of moisture and residual strains for parabolic distribution $\Delta W = W_{\max} \frac{x^n}{L_2^2}$. Was observed influence of distribution of properties and geometrical size of lumber on normal and tangential stresses.

Components of stresses for case, when initial moisture of lumber is higher than transition moisture content in the first period (bound moisture stands of from surface zones, and in central σ $W > W_m$) were found. Was obtained distribution of tangential and normal stresses of zone of lumber in drying process, which essentially depends on modulus of elasticity of wood along fibres.

Research of stress-strained wood state in drying process with taking into account of visco-elastic properties and peculiarities of strain regeneration in time. There are developed methods of synthesis and analysis, which are formalized with the aim of using of obtained experimental data of rheological wood behaviour allow quantitatively to set important in technology of wood drying regularities of distribution influence of hygroscopic moisture, its gradient, anisotroption of constant and changeable coefficients and internal sources of mass heat transfer, geometrical size of lumber on development of viscous-elastic and residual stresses in wood on different drying process stages.

Obtained dependences for determination of stress-strained state of drying wood are represented in structural form. For irregular drying process regime we have:

$$\sigma_B(X, F_0) = K^{t, \mu} \beta(U, X) \left(1/2 K I_m \left(1 + \varepsilon K o P n L u \left(1/3 - X^2 \right) - \frac{1}{\varepsilon K o} \sum_{n=1}^{\infty} \sum_{i=1}^2 C_m \left(1 - v_i^2 \right) \cos v_i \mu_n X - \frac{\sin v_i \mu_n}{v_i \mu_n} \right) \exp \left(- \mu_n^2 F_0 \right) \left(U_o - U_p \right); \right.$$

$$\sigma(X, F_0) = K^{t, \mu} \beta(U, X) \int_0^{\tau} \left(\partial U(X, F_0) / \partial \tau - \partial \bar{U}(\tau) / \partial \tau \right) \sum_{i=1}^2 S_1(U, t) K^0(\tau - \tau', U, t) d\tau' \quad (16)$$

Developed methods of synthesis and analysis allowed to set regularities of distribution of moistural and residual stresses in wood with constant and changeable coefficients of mass heat transfer for period of falling velocity of drying. We have considered cases of initial parabolic and cosinusoidal distribution of moisture content. For drying process of hardwood species moisture content in which can be described by cosinusoidal law, stress-strained state is determined by correlations:

$$\sigma_B(Y, \tau) = f(U, t) g(U_0, U_p, B_i) \exp \left(- \pi^2 B F / \left(4B + \pi^2 \right) \right); \quad (17)$$

$$\sigma(Y, \tau) = - \left(\exp(\tau/\tau) \right) / h(\tau, B, a') f(U, t) g(U_0, U_p, B_i) * \left(E(U, t) - E_T(U, t) \right) / E_T(U, t) \tau \left(\exp(h(\tau, B, a')) - 1 \right),$$

where

$$f(U,t) = F(U,t) / (1 + \beta_0 U_0) (U,t);$$

$$g(U_0, U_p, B_i) = 4(U_0 - U_p) \text{Bi}(\pi \cos \pi/2 - 1) / \pi (\pi^2 + 4 \text{Bi}^2);$$

$$h(\tau, B, a') = 1/\tau - \pi^2 a' B / R^2 (4B_i + \pi^2).$$

Influence of maximum stresses on cracking criterion was observed. , accepted as concerning gradient between local and mean moisture content and initial moisture content. For determining maximum-admissible senses in case of parabolic distribution of moisture content (regular regime) was obtained integral equation

$$\int_0^{\tau} K(\tau - \tau', U, t) K' d\tau' + K' = \frac{2 \sigma_{\max}(X, F_0) (1 + \beta_0 U_0)}{3 E(U, t) U_0 \beta(U, t) K} - \frac{\Delta t}{t_c - t_M} \text{GuPn} (1 + \tau (1 - \tau \exp(\tau/\tau))) \quad (18)$$

Stress-strained state of wood was studied with taking into account deepening of moisture evaporation zone in period of falling drying velocity. In connection with complication of temperature-moistural fields determining an assumption, that moisture content of moisture zone is constant, and in evaporation zone is equal to equilibrium sense was accepted. On base of analysis of results the presence of turns in stresses for different heat physical senses of deepening zone of moisture evaporation was showed, as well as augmentation of size of compressional stresses for some senses of evaporation zone in dry region of wood plate (without taking into account zone of tensional stresses).

On base of carried out researches basic regularities of development of viscous-elastic and residual stresses in wood during drying process were studied, with taking into account distribution of hygroscopic moisture, forms of moisture connection with material, physical and mechanical characteristics, conventional density (species) and geometrical size of lumber. Character of dynamics of residual stresses is stipulated by uneven development of moistural strains and creeping stains and reverse creeping dependently on forms of connection of moisture with material, and velocity of their development for constant change of moisture content is proportional to moisture gradient change. Peculiarity of forming of stresses in viscous-elastic region of straining is displacement of spectrum of maximum senses on lumber surfaces concerning maximum senses in central layers, caused by uneven distribution of level-by-level moisture and by dependence of time of maximum senses τ_{\max} reaching from relaxation time which for fixed senses of E_t and E_m are associated by correlation $\tau_{\max}(x) = 1,38 \tau_p^2 / \sigma_{\max} \gamma$ velocity of changing of moisture strains. Augmentation of conventional density sense causes growth of maximum stresses, especially on surface and in under surface lumber zones, as well as retardation of velocity of stresses relaxation. Specifically, for parameters of drying agent t , W% for wood with $\rho_0 = 550 \text{ kg/m}^3$ σ , σ' , is 0.18MPa to $XP_2=0$, and for $\rho_0 = 450 \text{ kg/m}^3$ analogic size is 1.37MPa.

Connection of components of stressed-strained state of drying wood with parameters of internal and external mass heat transfer. For quantitative description of connection interacting mass heat transfer flows on border of drying wood and moisture environment was analysed and described with taking into account behaviour peculiarities of bound moisture with material. It was taken into account that process of mass heat transfer in drying wood simultaneously goes in liquid, airvaporous and hard phases. Aiming at of concretization of thermodynamical components were made researches of them dependently on hygroscopic moisture of material, its structural properties with following normalization of sum of the flows, on general geometrical surface. Integral components of surfaces are determined from equality of volumetric and surface heterogeneities component of surface skeleton is a constant, and component of surface of liquid and gas-vaporous phases depend on moisture content of material.

Model of connection of stress-strained state of drying wood with parameters of external and internal mass heat transfer processes was synthesized, as well as dependences of heat-physical and mechanical characteristics of material with parameters of moistural air allow to determine maximum senses of parameters of drying agent directly at the expense of stresses, that do not exceed bound durability and provide integrity of material. Engineering correlations for φ and t_0 choice were obtained, with taking into account constant and changeable coefficients of mass heat transfer dependently on rheological properties of wood. Specifically, for parabolic distribution of moisture content in wood (period of constant velocity of drying):

$$t_c = \frac{0,13(238+t)\ln B}{1 + \frac{238+t}{1785} \ln B}, \quad \ln B = \frac{2a_m \rho_0 (1 + \beta_n U_0) \sigma}{E(U, t) K \text{Nu} D_p P_{nc}}. \quad (19)$$

For cosinusoidal distribution of moisture content in wood (period of falling-drying velocity) it was set:

$$\varphi = 0,018 * \frac{H_{os}^2 H_o^{4.8} U_{ost}}{V_s (1 - 3.1 \ln t/t_o)} * \quad (20)$$

$$* \left(\frac{\sigma_B(X, Fo)(1 + \beta_0 U_0) (U + (U - U) \cos X \pi/2)}{4(U_0 - U_p)(H(U, t) + K(U, t)) \frac{B_i \cos X \pi/2 - 2Bi/\pi}{4Bi + \pi^2} \exp\left(-\frac{4BiFo}{4Bi + \pi^2}\right)} + 1 \right) * \left(\frac{\lambda_u \varphi}{\rho_0} - \rho_s \frac{\alpha U l}{\text{Nu}_u} \left(\frac{1}{\rho_0} - \frac{1}{\rho_T} (U + (U - U) \cos X \pi/2) \right) \right)^{-1}.$$

Influence of parameters of moistural air on stress-strained state of wood in drying process was researched, with taking into account mechanical-sorptional creeping of material, caused by decreasing of moisture content in drying wood. A greater deformation change of wood in case of growth of relative moisture of air φ in compare of its decreasing was observed. Influence of temperature of moistural enviroment is more essential on development of residual strains and strains, caused by wood shrinkage.

Due to the research results a system of automatic regulation of drying agent parameters was developed, with taking into account stress-strained state of wood.

Symbols: ρ ó density; u ó moving; τ ó time; u_0, u_p, u, u_y ó moisture content: initial, equilibrium on surface, in the center; c ó thermal capacity; λ, ϑ, x ó coefficients of mass heat transfer: Poisson, Lame, filtration, kinematic viscosity, thermal conductions of dry wood; ε, r ó criterion and heat of phase transition; t^m, E_i, G_{xy} ó volumetric modulus of elasticity, modules of shift elasticity; $R(), K(), S1, S2$ ó functions of rheological behaviour (functions of relaxation, creeping and functionals of relaxation times); δ ó temperature gradients; $K_i, K_0, P_n, W, F_0, B_i, R_b$ ó criterions of Kirpichov, Kosovich, Posnov, Lykova, Fourier, Bio, Rebinder; c_m, ν, μ ó characteristic number; A, B ó parameters of approximation; δ ó Kronecker symbol.

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