[illegible] 517.956

[illegible]
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[illegible]

[illegible]

[illegible]

[illegible] A [illegible]

$$\frac{\partial u}{\partial t}+A(x,t,u)\frac{\partial u}{\partial x}=b(x,t,u), \qquad (1)$$

$(u,b$ – m [illegible]

$\lambda_i(x,t,u)$ [illegible] x,t,u, [illegible] (1) [illegible] [1].

[illegible] [2, 3]

$$\frac{\partial z}{\partial t}+\Lambda(x,t,z)\frac{\partial z}{\partial x}=F(x,t,z),\ \Lambda=diag(\lambda_1,...,\lambda_m), \qquad (2)$$

[illegible]

$$G(x,t,u)\frac{\partial u}{\partial t}+\Lambda(x,t,u)G(x,t,u)\frac{\partial u}{\partial x}=R(x,t,u), \qquad (3)$$

[illegible] G [illegible] A. [illegible] (2) [illegible] (3) [illegible] [1].

[illegible] [1, 4], [illegible] (3) [illegible] (2). [illegible] (1) [illegible] (3) [illegible]

$$\begin{cases}\dfrac{\partial P}{\partial t}+\Lambda\dfrac{\partial P}{\partial x}=\Omega(x,t,u,P,Q),\\ \dfrac{\partial Q}{\partial t}+\Lambda\dfrac{\partial Q}{\partial x}=\Psi(x,t,u,P,Q),\\ \dfrac{\partial u}{\partial t}=G^{-1}Q,\ \dfrac{\partial u}{\partial x}=G^{-1}P,\end{cases} \qquad (4)$$

[illegible] $p=\dfrac{\partial u}{\partial x},\ q=\dfrac{\partial u}{\partial t},\ P=Gp,\ Q=Gq.$

[illegible] (4) [illegible] [1, 5]. [illegible] [5 - 10].

[illegible] [11, 12].

[illegible] $G=\{(x,t):0<t<T,\ -kt<x<kt+l\}$, T,k,l [illegible]

$$\frac{\partial u}{\partial t}+\Lambda(x,t)\frac{\partial u}{\partial x}=f(x,t,u,v),$$
$$\frac{\partial v}{\partial x}=g(x,t,u,v), \qquad (5)$$

[illegible] $u=(u_1,...,u_m),\ v=(v_1,...,v_n),\ f=(f_1,...,f_m),\ g=(g_1,...,g_n),\ \Lambda(x,t)=diag(\lambda_1(x,t),...,\lambda_m(x,t))$.

[illegible] (5) [illegible]

$$u(x,0)=q(x),\ (q=(q_1,...,q_m)),\ 0\le x\le l \tag{6}$$

[illegible] $0\le t\le T$:

$$\begin{aligned} &u_i(-kt,t)=\gamma_i^0(t,u(-kt,t)), \qquad i\in I_0,\\ &u_i(l+kt,t)=\gamma_i^l(t,(u(l+kt,t)),\ i\in I_l,\\ &v(-kt,t)=\psi(t,u(-kt,t)),\ (\psi=(\psi_1,...,\psi_n)),\end{aligned} \tag{7}$$

[illegible] I_0, I_l [illegible]:

$I_0=\{i\in\{1,\dots,m\}:\lambda_i(0,t)>-k\},\ I_l=\{i\in\{1,\dots,m\}:\lambda_i(l,t)<k\}$.

[illegible] I_0, I_l [illegible] r_0, r_l [illegible] γ_i^0, ψ_j [illegible] u_i [illegible] $i\in I_0$ [illegible] γ_i^l [illegible] u_i [illegible] $i\in I_l$ [illegible]

$$(\lambda_i(-kt,t)+k)(\lambda_i(l+kt,t)-k)\ne 0,\quad t\in[0,T],\quad i\in\{1,\dots,m\}.$$

[illegible]

$$f:\overline{G}\times R^{m+n}\to R^m,\ g:\overline{G}\times R^{m+n}\to R^n, \lambda_i:\overline{G}\to R^m,\ q:[0,l]\to R^m,$$

$$\gamma^0:[0,T]\times R^{r_l}\to R^{r_0},\ \gamma^l:[0,T]\times R^{r_0}\to R^{r_l},\ \psi:[0,T]\times R^{r_l}\to R^n$$

[illegible] λ_i [illegible] x.

[illegible]. [illegible] $\varphi_i(t;x_0,t_0),\quad i\in\{1,\dots,m\}$ [illegible]

$$\frac{dx}{dt}=\lambda_i(x,t),\ \ x(t_0)=x_0$$

[illegible] (5). [illegible] $t=t_0$. [illegible] $\chi_i(x_0,t_0)$ [illegible] t, [illegible] $\overline{G}$ [illegible] $x=\varphi_i(t;x_0,t_0)$.

$$G_q^i=\{(x,t)\in G:\chi_i(x,t)=0\},\ i\in\{1,\dots,m\},$$

[illegible]: $G_0^i=\{(x,t)\in G:\chi_i(x,t)>0,\ \varphi_i(\chi_i(x,t);x,t)=-k\chi_i(x,t)\}, i\in I_0$,

$$G_l^i=\{(x,t)\in G:\chi_i(x,t)>0,\ \varphi_i(\chi_i(x,t);x,t)=l+k\chi_i(x,t)\}, i\in I_l.$$

[illegible] (5)-(7) [illegible] [12]:

$$\begin{aligned} &u_i(x,t)=F_i[w](x,t)+\\ &+\int_{\chi_i(x,t)}^{t} f_i(\varphi_i(\tau;x,t),\tau,u(\varphi_i(\tau;x,t),\tau),v(\varphi_i(\tau;x,t),\tau))d\tau,\\ &i\in\{1,\dots,m\},\end{aligned} \tag{8}$$

$$\begin{aligned} &v_j(x,t)=\psi_j(t,u(-kt,t))+\int_{-kt}^{x} g_j(y,t,u(y,t),v(y,t))dy,\\ &j\in\{1,\dots,n\},\end{aligned} \tag{9}$$

[illegible]

$$F_i[w](x,t)=\begin{cases} q_i(\varphi_i(0;x,t)),(x,t)\in G_q^i,\\ \gamma_i^0(\chi_i(x,t),u(-k\chi_i(x,t),\chi_i(x,t))),(x,t)\in G_0^i,\\ \gamma_i^l(\chi_i(x,t),u(l+k\chi_i(x,t),\chi_i(x,t))),(x,t)\in G_l^i.\end{cases} \tag{10}$$

[illegible]. [illegible] (5)-(7), [illegible] (u,v), [illegible] $C(\overline{G})$ [illegible] (8)-(9).

[illegible].

[illegible]:

1) $\lambda, f, g, q, \psi, \gamma^1, \gamma^2$ [illegible];
2) [illegible] λ [illegible] $\overline{G}$ [illegible] x;
3) [illegible] $f, g, q, \psi, \gamma^1, \gamma^2$ [illegible] u, v [illegible];
4) [illegible]

$$(\lambda_i(-kt,t)+k)(\lambda_i(l+kt,t)-k) \neq 0,\ t \in [0,T],\ i \in \{1,\dots,m\};$$

5) [illegible]

$$q_i(0) = \gamma_i^0(0,q(0)), \quad i \in I_0,$$
$$q_i(l) = \gamma_i^l(0,q(l)), \quad i \in I_l.$$

[illegible] (5)-(7) [illegible] $\overline{G}$.

[illegible]. [illegible] Q, [illegible] $w=(u,v)$ [illegible] $C(\overline{G})$, [illegible] $u_i(0,0)=q_i(0), i \in I_0$ [illegible] $u_i(l,0)=q_i(l), i \in I_l$. [illegible]

$$\rho(w^1,w^2) = \max\{\max_{i,x,t} |u_i^1(x,t)-u_i^2(x,t)|\,\alpha_i(x,t)e^{-at},$$
$$\max_{i,x,t}|v_i^1(x,t)-v_i^2(x,t)|\,\beta_i(x,t)e^{-at}\}, \tag{11}$$

[illegible] $a>0$ [illegible] α_i, β_i [illegible].

[illegible] Q [illegible] $A=\left(A^1,A^2\right)$, [illegible] $A[w]=(A^1[u,v],A^2[u,v])$, [illegible] A^1 [illegible] A^2 [illegible] (8), (9), [illegible]

$$A_i^1[w](x,t) = F_i[w](x,t) +$$
$$+\int_{\chi_i(x,t)}^{t} f_i(\varphi_i(\tau;x,t),\tau,u(\varphi_i(\tau;x,t),\tau),v(\varphi_i(\tau;x,t),\tau))d\tau,$$
$$i \in \{1,\dots,m\},$$
$$A_j^2[w](x,t) = \psi_j(t,u(-kt,t)) + \int_{-kt}^{x} g_j(y,t,u(y,t),v(y,t))dy,$$
$$j \in \{1,\dots,n\}.$$

[illegible] (5)-(7) [illegible] A [illegible] Q. [illegible] A [illegible] $\varphi_i(\tau;x,t)$ [illegible] $\chi_i(x,t)$.

[illegible] L [illegible] $f, g, \psi, \gamma^0, \gamma^l$, [illegible]: $|f_i(x,t,u^1,v^1)-f_i(x,t,u^2,v^2)| \le L\max\{\max_{j,x,t}|u_j^1-u_j^2|, \max_{j,x,t}|v_j^1-v_j^2|\}$, [illegible].

[illegible] $w^1 \in Q, w^2 \in Q$. [illegible] (11) [illegible] i, x, t [illegible]

$$|u_i^1(x,t)-u_i^2(x,t)|\le\frac{\rho(w^1,w^2)}{\alpha_i(x,t)}e^{at},$$

$$|v_i^1(x,t)-v_i^2(x,t)|\le\frac{\rho(w^1,w^2)}{\beta_j(x,t)}e^{at}.$$

[illegible] A, [illegible] $w^1,w^2\in Q$, [illegible] (10) [illegible]

$$|F_i[w^1](x,t)-F_i[w^2](x,t)|\le$$

$$\le\begin{cases} L\max\limits_{j\notin I_0}\dfrac{\rho(w^1,w^2)}{\alpha_j(-k\chi_i(x,t),\chi_i(x,t))}e^{a\chi_i(x,t)}, & (x,t)\in G_0^i,\\ L\max\limits_{j\notin I_l}\dfrac{\rho(w^1,w^2)}{\alpha_j(l+k\chi_i(x,t),\chi_i(x,t))}e^{a\chi_i(x,t)}, & (x,t)\in G_l^i. \end{cases} \qquad (12)$$

[illegible] (12), [illegible] (5) [illegible] $i\in I_0$ [illegible] $\chi_i(x,t)\le\dfrac{-x+\Lambda t}{\Lambda+k}$, [illegible] $\Lambda=\max\limits_{i,x,t}|\lambda_i(x,t)|$. [illegible] $i\in I_l$ [illegible] $\chi_i(x,t)\le\dfrac{x+\Lambda t-l}{\Lambda+k}$.

[illegible] A^1 [illegible]

$$\left|\left(A_i^1\left[w^1\right](x,t)\right)-\left(A_i^1\left[w^2\right](x,t)\right)\right|\alpha_i(x,t)e^{-at}\le$$

$$\le L\max\left\{\max_{\substack{i\in I_0,\\ j\notin I_0,\tau}}\frac{\alpha_i(x,t)e^{a\left(\frac{-x+\Lambda t}{\Lambda+k}-t\right)}}{\alpha_j(-k\tau,\tau)},\ \max_{\substack{i\in I_l,\\ j\notin I_l,\tau}}\frac{\alpha_i(x,t)e^{a\left(\frac{x+\Lambda t}{\Lambda+k}-t\right)}}{\alpha_j(l+k\tau,\tau)}\right\}\rho\left(w^1,w^2\right)+$$

$$+\int_0^t e^{a(\sigma-t)}d\sigma\, L\max\left\{\max_{i,j,y,\tau}\frac{\alpha_i(x,t)}{\alpha_j(y,\tau)},\ \max_{i,j,y,\tau}\frac{\alpha_i(x,t)}{\beta_j(y,\tau)}\right\}\rho\left(w^1,w^2\right)\le$$

$$\le L\max\left\{\max_{\substack{i\in I_0,\\ j\notin I_0,\tau}}\frac{\alpha_i(x,t)e^{a\frac{-x-kt}{\Lambda+k}}}{\alpha_j(-k\tau,\tau)},\ \max_{\substack{i\in I_l,\\ j\notin I_l,\tau}}\frac{\alpha_i(x,t)e^{a\frac{x-kt-l}{\Lambda+k}}}{\alpha_j(l+k\tau,\tau)}\right\}\rho\left(w^1,w^2\right)+$$

$$+\frac{L}{a}\max\left\{\max_{i,j,y,\tau}\frac{\alpha_i(x,t)}{\alpha_j(y,\tau)},\ \max_{i,j,y,\tau}\frac{\alpha_i(x,t)}{\beta_j(y,\tau)}\right\}\rho\left(w^1,w^2\right). \qquad (13)$$

[illegible] A^2, [illegible]

$$|A_i^2[w^1](x,t)-A_i^2[w^2](x,t)|\beta_i(x,t)e^{-at}\le(L\max_{i,j\notin I_0}\frac{\beta_i(x,t)}{\alpha_j(-kt,t)}+$$

$$+\int_{-kt}^{x}L\max\{\max_{i,j}\frac{\beta_i(x,t)}{\alpha_j(y,t)},\max_{i,j}\frac{\beta_i(x,t)}{\beta_j(y,t)}\}dy)\rho(w^1,w^2). \qquad (14)$$

[illegible] (11) [illegible] (13) [illegible] (14) [illegible]:

$$\rho(A[w^1],A[w^2]) \le \max_{(x,t)\in\bar G}\left\{ L\max\{\max_{\substack{i\in I_0,\\ j\notin I_0,\tau}} \frac{\alpha_i(x,t)e^{a\frac{-x-kt}{\Lambda+k}}}{\alpha_j(-k\tau,\tau)},\right.$$

$$\left.\max_{\substack{i\in I_l,\\ j\notin I_l,\tau}} \frac{\alpha_i(x,t)e^{a\frac{x-kt-l}{\Lambda+k}}}{\alpha_j(l+k\tau,\tau)}\right\} + \frac{L}{a}\max\left\{\max_{i,j,y,\tau}\frac{\alpha_i(x,t)}{\alpha_j(y,\tau)},\max_{i,j,y,\tau}\frac{\alpha_i(x,t)}{\beta_j(y,\tau)}\right\}+$$

$$\left.+L\max_{i,j\notin I_0}\frac{\beta_i(x,t)}{\alpha_j(-kt,t)}+\int_{-kt}^{x} L\max\{\max_{i,j}\frac{\beta_i(x,t)}{\alpha_j(y,t)},\max_{i,j}\frac{\beta_i(x,t)}{\beta_j(y,t)}\}dy\right\}\rho(w^1,w^2).$$

[illegible] α_i, β_i, [illegible] A [illegible]

$$\alpha_i(x,t)=\begin{cases} e^{p(kt+x)(kt-x+l)}, & i\in I_0, i\in I_l,\\ e^{p(kt+x)}, & i\in I_0, i\notin I_l,\\ e^{p(kt-x+l)}, & i\notin I_0, i\in I_l,\\ e^{p(2kt+l)}, & i\notin I_0, i\notin I_l,\end{cases}$$

$$\beta_i(x,t)=\varepsilon e^{-p(kt+x)}.$$

[illegible] $\mu=\dfrac{1}{\Lambda+k}$ [illegible]

$$p\le a\mu,\ pl-a\mu\le -2pkT.$$

[illegible] $\bar G$, [illegible]

$$\max_{(x,t)\in\bar G}\max_{\substack{i\in I_0,\\ j\notin I_0,\tau}}\frac{\alpha_i(x,t)e^{a\mu(-x-kt)}}{\alpha_j(-k\tau,\tau)}=\max_{(x,t)\in\bar G}\max_{i\in I_0}\frac{\alpha_i(x,t)e^{a\mu(-x-kt)}}{e^{pl}}=$$

$$=\max_{(x,t)\in\bar G}\max\left\{e^{p(kt+x)(kt-x+l)},e^{p(kt+x)}\right\}e^{a\mu(-x-kt)-pl}=e^{-pl},$$

$$\max_{(x,t)\in\bar G}\max_{\substack{i\in I_l,\\ j\notin I_l,\tau}}\frac{\alpha_i(x,t)e^{a\mu(x-kt-l)}}{\alpha_j(l+k\tau,\tau)}=\max_{(x,t)\in\bar G}\max_{i\in I_l}\frac{\alpha_i(x,t)e^{a\mu(x-kt-l)}}{e^{pl}}=$$

$$=\max_{(x,t)\in\bar G}\max\left\{e^{p(kt+x)(kt-x+l)},e^{p(kt-x-l)}\right\}e^{a\mu(x-kt-l)-pl}=e^{-pl}.$$

[illegible]

$$\max_{(x,t)\in\bar G}\max_{i,j\notin I_0}\frac{\beta_i(x,t)}{\alpha_j(-kt,t)}=\max_{(x,t)\in\bar G}\frac{\varepsilon e^{-p(kt+x)}}{e^{p(2kt+l)}}=\max_{(x,t)\in\bar G}\varepsilon e^{-3pkt-px-pl}=$$

$$=\max_t v\varepsilon e^{-2pkt+pl}=\varepsilon e^{-pl}.$$

[illegible]

$$\max_{(x,t)\in\bar G}\int_{-kt}^{x}\max\{\max_{i,j}\frac{\beta_i(x,t)}{\alpha_j(y,t)};\max_{i,j}\frac{\beta_i(x,t)}{\beta_j(y,t)}\}dy\le$$

$$\max_{(x,t)\in\bar G}\int_{-kt}^{x}\max_{i,j}\frac{\beta_i(x,t)}{\alpha_j(y,t)}dy+\max_{(x,t)\in\bar G}\int_{-kt}^{x}\max_{i,j}\frac{\beta_i(x,t)}{\beta_j(y,t)}dy.$$

[illegible]

$$\max_{(x,t)\in\bar G}\int_{-kt}^{x}\max_{i,j}\frac{\beta_i(x,t)}{\alpha_j(y,t)}dy=\max_{(x,t)\in\bar G}\int_{-kt}^{x}\varepsilon e^{-p(kt+x)}dy\le$$

$$\le\max_{(x,t)\in\bar G}\{\varepsilon e^{-p(kt+x)}(x+kt)\}=\varepsilon(l+2kT),$$

$$\max_{(x,t)\in \overline{G}} \int_{-kt}^{x} \max_{i,j} \frac{\beta_i(x,t)}{\beta_j(y,t)} dy = \max_{(x,t)\in \overline{G}} \int_{-kt}^{x} \frac{\varepsilon e^{-p(kt+x)}}{\varepsilon e^{-p(kt+y)}} dy =$$

$$= \max_{(x,t)\in \overline{G}} \int_{-kt}^{x} e^{p(y-x)} dy = \max_{(x,t)\in \overline{G}} \frac{1}{p}(1-e^{-p(kt+x)}) \le \frac{1}{p}.$$

[illegible],

$$\max_{(x,t)\in \overline{G}} \int_{-kt}^{x} \max\{\max_{i,j} \frac{\beta_i(x,t)}{\alpha_j(y,t)}; \max_{i,j} \frac{\beta_i(x,t)}{\beta_j(y,t)}\} dy \le \varepsilon(l+2kT) + \frac{1}{p}.$$

[illegible]

$$\rho(A[w^1], A[w^2]) \le (Le^{-pl} + \frac{L}{a}\max\left\{\max_{i,j,y,\tau} \frac{\alpha_i(x,t)}{\alpha_j(y,\tau)}, \max_{i,j,y,\tau} \frac{\alpha_i(x,t)}{\beta_j(y,\tau)}\right\} +$$

$$+ L\varepsilon + L(\varepsilon(l+2kT) + \frac{1}{p}))\rho(w^1, w^2).$$

[illegible] p^*, ε^* [illegible]

$$Le^{-pl} + L\varepsilon + L(\varepsilon(l+2kT) + \frac{1}{p}) < \frac{1}{2},$$

[illegible] α_i^*, β_i^* [illegible] α_i, β_i [illegible] $p = p^*, \varepsilon = \varepsilon^*$. [illegible]

$$M = \max_{(x,t)\in \overline{G}} \left\{\max_{i,j,y,\tau} \frac{\alpha_i^*(x,t)}{\alpha_j^*(y,\tau)}, \max_{i,j,y,\tau} \frac{\alpha_i^*(x,t)}{\beta_j^*(y,\tau)}\right\}.$$

[illegible] a^*, [illegible]

$$p^* \le a^*\mu, \quad p^*l - a^*\mu \le -2p^*kT, \quad \frac{LM}{a^*} < \frac{1}{2}.$$

[illegible] A [illegible] $Q^* = Q$ [illegible] $\alpha_i = \alpha_i^*, \beta_i = \beta_i^*$ [illegible] $a = a^*$.

[illegible] A [illegible] Q^*. [illegible] (5)-(7).

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