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$$\frac{\partial u}{\partial t} + A(x,t,u) \frac{\partial u}{\partial x} = b(x,t,u), \quad (1)$$

( $u, b$  ó  $m$  -  $\lambda_i(x,t,u)$  -  $x, t, u$ , (1) [1].

$$\frac{\partial z}{\partial t} + \Lambda(x,t,z) \frac{\partial z}{\partial x} = F(x,t,z), \quad \Lambda = \text{diag}(\lambda_1, \dots, \lambda_m), \quad (2)$$

$$G(x,t,u) \frac{\partial u}{\partial t} + \Lambda(x,t,u) G(x,t,u) \frac{\partial u}{\partial x} = R(x,t,u), \quad (3)$$

$G$  ó  $A$ . (2) (3) [1]. [1, 4], (3) (2). (1)

$$\begin{cases} \frac{\partial P}{\partial t} + \Lambda \frac{\partial P}{\partial x} = \Omega(x,t,u,P,Q), \\ \frac{\partial Q}{\partial t} + \Lambda \frac{\partial Q}{\partial x} = \Psi(x,t,u,P,Q), \\ \frac{\partial u}{\partial t} = G^{-1}Q, \quad \frac{\partial u}{\partial x} = G^{-1}P, \end{cases} \quad (4)$$

$$p = \frac{\partial u}{\partial x}, \quad q = \frac{\partial u}{\partial t}, \quad P = Gp, \quad Q = Gq. \quad (4)$$

[1, 5]. [5 - 10].

[11, 12].

$$G = \{(x,t) : 0 < t < T, -kt < x < kt + l\}, \quad T, k, l -$$

$$\begin{cases} \frac{\partial u}{\partial t} + \Lambda(x,t) \frac{\partial u}{\partial x} = f(x,t,u,v), \\ \frac{\partial v}{\partial x} = g(x,t,u,v), \end{cases} \quad (5)$$

$$u = (u_1, \dots, u_m), v = (v_1, \dots, v_n), f = (f_1, \dots, f_m), g = (g_1, \dots, g_n), \Lambda(x, t) = \text{diag}(\lambda_1(x, t), \dots, \lambda_m(x, t)). \quad (5)$$

$$u(x, 0) = q(x), (q = (q_1, \dots, q_m)), 0 \leq x \leq l \quad (6)$$

$$0 \leq t \leq T :$$

$$\begin{aligned} u_i(-kt, t) &= \gamma_i^0(t, u(-kt, t)), \quad i \in I_0, \\ u_i(l+kt, t) &= \gamma_i^l(t, u(l+kt, t)), \quad i \in I_l, \\ v(-kt, t) &= \psi(t, u(-kt, t)), \quad (\psi = (\psi_1, \dots, \psi_n)), \end{aligned} \quad (7)$$

$$I_0, I_l \text{ } \delta \quad , \quad : \\ I_0 = \{i \in \{1, \dots, m\} : \lambda_i(0, t) > -k\}, I_l = \{i \in \{1, \dots, m\} : \lambda_i(l, t) < k\}.$$

$$I_0, I_l \quad , \quad r_0, r_l \quad . \\ \gamma_i^0, \psi_j \text{ } \delta \quad u_i \quad i \in I_0 \quad \gamma_i^l \text{ } \delta \quad u_i \quad i \in I_l \\ (\lambda_i(-kt, t) + k)(\lambda_i(l+kt, t) - k) \neq 0, \quad t \in [0, T], \quad i \in \{1, \dots, m\}.$$

$$\begin{aligned} f : \bar{G} \times R^{m+n} &\rightarrow R^m, g : \bar{G} \times R^{m+n} \rightarrow R^n, \lambda_i : \bar{G} \rightarrow R^m, q : [0, l] \rightarrow R^m, \\ \gamma^0 : [0, T] \times R^{r_l} &\rightarrow R^{r_0}, \gamma^l : [0, T] \times R^{r_0} \rightarrow R^{r_l}, \psi : [0, T] \times R^{r_l} \rightarrow R^n \\ , \quad \lambda_i \text{ } \delta \quad , \quad x. \\ \varphi_i(t; x_0, t_0), \quad i \in \{1, \dots, m\} \end{aligned}$$

$$\frac{dx}{dt} = \lambda_i(x, t), \quad x(t_0) = x_0$$

(5).

$$t = t_0. \quad \chi_i(x_0, t_0) \quad t ,$$

$$\bar{G} \quad , \quad x = \varphi_i(t; x_0, t_0).$$

$$G_q^i = \{(x, t) \in G : \chi_i(x, t) = 0\}, i \in \{1, \dots, m\},$$

$$: G_0^i = \{(x, t) \in G : \chi_i(x, t) > 0, \varphi_i(\chi_i(x, t); x, t) = -k\chi_i(x, t)\}, i \in I_0,$$

$$G_l^i = \{(x, t) \in G : \chi_i(x, t) > 0, \varphi_i(\chi_i(x, t); x, t) = l + k\chi_i(x, t)\}, i \in I_l.$$

(5)-(7)

[12]:

$$u_i(x, t) = F_i[w](x, t) +$$

$$+ \int_{z_i(x, t)}^t f_i(\varphi_i(\tau; x, t), \tau, u(\varphi_i(\tau; x, t), \tau), v(\varphi_i(\tau; x, t), \tau)) d\tau, \quad (8)$$

$$i \in \{1, \dots, m\},$$

$$v_j(x, t) = \psi_j(t, u(-kt, t)) + \int_{-kt}^x g_j(y, t, u(y, t), v(y, t)) dy, \quad (9)$$

$$j \in \{1, \dots, n\},$$

$$F_i[w](x, t) = \begin{cases} q_i(\varphi_i(0; x, t)), (x, t) \in G_q^i, \\ \gamma_i^0(\chi_i(x, t), u(-k\chi_i(x, t), \chi_i(x, t))), (x, t) \in G_0^i, \\ \gamma_i^l(\chi_i(x, t), u(l+k\chi_i(x, t), \chi_i(x, t))), (x, t) \in G_l^i. \end{cases} \quad (10)$$

- (5)-(7),  
 $(u, v), C(\bar{G})$   
 (8)-(9).
- 1)  $\lambda, f, g, q, \psi, \gamma^1, \gamma^2$  ;
  - 2)  $\lambda \in \bar{G}$  ;
  - 3)  $f, g, q, \psi, \gamma^1, \gamma^2$  ;
  - 4)  $(\lambda_i(-kt, t) + k)(\lambda_i(l + kt, t) - k) \neq 0, t \in [0, T], i \in \{1, \dots, m\}$ ;
  - 5)  $q_i(0) = \gamma_i^0(0, q(0)), i \in I_0,$   
 $q_i(l) = \gamma_i^l(0, q(l)), i \in I_l.$

(5)-(7)  $\bar{G}$ .

$Q,$

$w = (u, v) \in C(\bar{G}), u_i(0, 0) = q_i(0), i \in I_0$   
 $u_i(l, 0) = q_i(l), i \in I_l.$

$$\rho(w^1, w^2) = \max\{\max_{i,x,t} |u_i^1(x,t) - u_i^2(x,t)| \alpha_i(x,t) e^{-at}, \max_{i,x,t} |v_i^1(x,t) - v_i^2(x,t)| \beta_i(x,t) e^{-at}\}, \quad (11)$$

$a > 0$

$Q$   $A = (A^1, A^2),$

$A[w] = (A^1[u, v], A^2[u, v]),$   $A^1 A^2$   
 (8), (9),

$$A_i^1[w](x, t) = F_i[w](x, t) + \int_{z_i(x,t)}^t f_i(\varphi_i(\tau; x, t), \tau, u(\varphi_i(\tau; x, t), \tau), v(\varphi_i(\tau; x, t), \tau)) d\tau,$$

$$i \in \{1, \dots, m\},$$

$$A_j^2[w](x, t) = \psi_j(t, u(-kt, t)) + \int_{-kt}^x g_j(y, t, u(y, t), v(y, t)) dy,$$

$$j \in \{1, \dots, n\}.$$

(5)-(7)

$A$   $Q.$

$A$   $\varphi_i(\tau; x, t) \chi_i(x, t).$

$L$   $f, g, \psi, \gamma^0, \gamma^l,$

$: |f_i(x, t, u^1, v^1) - f_i(x, t, u^2, v^2)| \leq L \max\{\max_{j,x,t} |u_j^1 - u_j^2|, \max_{j,x,t} |v_j^1 - v_j^2|\},$

$w^1 \in Q, w^2 \in Q. \quad (11) \quad i, x, t$

$$|u_i^1(x, t) - u_i^2(x, t)| \leq \frac{\rho(w^1, w^2)}{\alpha_i(x, t)} e^{at},$$

$$|v_i^1(x, t) - v_i^2(x, t)| \leq \frac{\rho(w^1, w^2)}{\beta_j(x, t)} e^{at}.$$

(10)

$$|F_i[w^1](x, t) - F_i[w^2](x, t)| \leq \begin{cases} L \max_{j \notin I_0} \frac{\rho(w^1, w^2)}{\alpha_j(-k\chi_i(x, t), \chi_i(x, t))} e^{a\chi_i(x, t)}, & (x, t) \in G_0^i, \\ L \max_{j \in I_1} \frac{\rho(w^1, w^2)}{\alpha_j(l + k\chi_i(x, t), \chi_i(x, t))} e^{a\chi_i(x, t)}, & (x, t) \in G_1^i. \end{cases} \tag{12}$$

$w^1, w^2 \in Q,$

(12), (5)  $i \in I_0$

$$\chi_i(x, t) \leq \frac{-x + \Lambda t}{\Lambda + k}, \quad \Lambda = \max_{i, x, t} |\lambda_i(x, t)|,$$

$$i \in I_1 \quad \chi_i(x, t) \leq \frac{x + \Lambda t - l}{\Lambda + k}.$$

$$\begin{aligned} & |(A_i^1[w^1](x, t) - A_i^1[w^2](x, t))\alpha_i(x, t)e^{-at}| \leq \\ & \leq L \max \left\{ \max_{\substack{i \in I_0, \\ j \notin I_0, \tau}} \frac{\alpha_i(x, t)e^{a\left(\frac{-x+\Lambda t}{\Lambda+k}-\tau\right)}}{\alpha_j(-k\tau, \tau)}, \max_{\substack{i \in I_1, \\ j \in I_1, \tau}} \frac{\alpha_i(x, t)e^{a\left(\frac{x+\Lambda t}{\Lambda+k}-\tau\right)}}{\alpha_j(l+k\tau, \tau)} \right\} \rho(w^1, w^2) + \\ & + \int_0^t e^{a(\sigma-t)} d\sigma L \max \left\{ \max_{i, j, y, \tau} \frac{\alpha_i(x, t)}{\alpha_j(y, \tau)}, \max_{i, j, y, \tau} \frac{\alpha_i(x, t)}{\beta_j(y, \tau)} \right\} \rho(w^1, w^2) \leq \\ & \leq L \max \left\{ \max_{\substack{i \in I_0, \\ j \notin I_0, \tau}} \frac{\alpha_i(x, t)e^{a\left(\frac{-x-kt}{\Lambda+k}-\tau\right)}}{\alpha_j(-k\tau, \tau)}, \max_{\substack{i \in I_1, \\ j \in I_1, \tau}} \frac{\alpha_i(x, t)e^{a\left(\frac{x-kt-l}{\Lambda+k}-\tau\right)}}{\alpha_j(l+k\tau, \tau)} \right\} \rho(w^1, w^2) + \\ & + \frac{L}{a} \max \left\{ \max_{i, j, y, \tau} \frac{\alpha_i(x, t)}{\alpha_j(y, \tau)}, \max_{i, j, y, \tau} \frac{\alpha_i(x, t)}{\beta_j(y, \tau)} \right\} \rho(w^1, w^2). \end{aligned} \tag{13}$$

$$|A_i^2[w^1](x, t) - A_i^2[w^2](x, t)|\beta_i(x, t)e^{-at} \leq (L \max_{i, j \in I_0} \frac{\beta_i(x, t)}{\alpha_j(-kt, t)} + \int_{-kt}^x L \max \{ \max_{i, j} \frac{\beta_i(x, t)}{\alpha_j(y, t)}, \max_{i, j} \frac{\beta_i(x, t)}{\beta_j(y, t)} \} dy) \rho(w^1, w^2). \tag{14}$$

(11) (13) (14) :

$$\rho(A[w^1], A[w^2]) \leq \max_{(x,t) \in \bar{G}} \left\{ L \max_{\substack{i \in I_0, \\ j \notin I_0, \tau}} \frac{\alpha_i(x,t) e^{\frac{-x-kt}{\Lambda+k}}}{\alpha_j(-k\tau, \tau)}, \right. \\ \left. \max_{\substack{i \in I_1, \\ j \notin I_1, \tau}} \frac{\alpha_i(x,t) e^{\frac{x-kt-l}{\Lambda+k}}}{\alpha_j(l+k\tau, \tau)} \right\} + \frac{L}{a} \max \left\{ \max_{i,j,y,\tau} \frac{\alpha_i(x,t)}{\alpha_j(y,\tau)}, \max_{i,j,y,\tau} \frac{\alpha_i(x,t)}{\beta_j(y,\tau)} \right\} + \\ + L \max_{i,j \notin I_0} \frac{\beta_i(x,t)}{\alpha_j(-kt, t)} + \int_{-kt}^x L \max \left\{ \max_{i,j} \frac{\beta_i(x,t)}{\alpha_j(y,t)}, \max_{i,j} \frac{\beta_i(x,t)}{\beta_j(y,t)} \right\} dy \Big\} \rho(w^1, w^2).$$

$\alpha_i, \beta_i, \quad A$

$$\alpha_i(x,t) = \begin{cases} e^{p(kt+x)(kt-x+l)}, & i \in I_0, i \in I_1, \\ e^{p(kt+x)}, & i \in I_0, i \notin I_1, \\ e^{p(kt-x+l)}, & i \notin I_0, i \in I_1, \\ e^{p(2kt+l)}, & i \notin I_0, i \notin I_1, \end{cases}$$

$$\beta_i(x,t) = \varepsilon e^{-p(kt+x)}.$$

$$\mu = \frac{1}{\Lambda+k}$$

$$p \leq a\mu, \quad pl - a\mu \leq -2pkT.$$

$$\begin{aligned} & \max_{(x,t) \in \bar{G}} \max_{\substack{i \in I_0, \\ j \notin I_0, \tau}} \frac{\alpha_i(x,t) e^{a\mu(-x-kt)}}{\alpha_j(-k\tau, \tau)} = \max_{(x,t) \in \bar{G}} \max_{i \in I_0} \frac{\alpha_i(x,t) e^{a\mu(-x-kt)}}{e^{pl}} = \\ & = \max_{(x,t) \in \bar{G}} \max \left\{ e^{p(kt+x)(kt-x+l)}, e^{p(kt+x)} \right\} e^{a\mu(-x-kt)-pl} = e^{-pl}, \\ & \max_{(x,t) \in \bar{G}} \max_{\substack{i \in I_1, \\ j \notin I_1, \tau}} \frac{\alpha_i(x,t) e^{a\mu(x-kt-l)}}{\alpha_j(l+k\tau, \tau)} = \max_{(x,t) \in \bar{G}} \max_{i \in I_1} \frac{\alpha_i(x,t) e^{a\mu(x-kt-l)}}{e^{pl}} = \\ & = \max_{(x,t) \in \bar{G}} \max \left\{ e^{p(kt+x)(kt-x+l)}, e^{p(kt-x+l)} \right\} e^{a\mu(x-kt-l)-pl} = e^{-pl}. \\ & \max_{(x,t) \in \bar{G}} \max_{i,j \notin I_0} \frac{\beta_i(x,t)}{\alpha_j(-kt, t)} = \max_{(x,t) \in \bar{G}} \frac{\varepsilon e^{-p(kt+x)}}{e^{p(2kt+l)}} = \max_{(x,t) \in \bar{G}} \varepsilon e^{-3pkt-px-pl} = \\ & = \max_t \varepsilon e^{-2pkt+pl} = \varepsilon e^{-pl}. \end{aligned}$$

$$\begin{aligned} & \max_{(x,t) \in \bar{G}} \int_{-kt}^x \max \left\{ \max_{i,j} \frac{\beta_i(x,t)}{\alpha_j(y,t)}, \max_{i,j} \frac{\beta_i(x,t)}{\beta_j(y,t)} \right\} dy \leq \\ & \max_{(x,t) \in \bar{G}} \int_{-kt}^x \max_{i,j} \frac{\beta_i(x,t)}{\alpha_j(y,t)} dy + \max_{(x,t) \in \bar{G}} \int_{-kt}^x \max_{i,j} \frac{\beta_i(x,t)}{\beta_j(y,t)} dy. \\ & \max_{(x,t) \in \bar{G}} \int_{-kt}^x \max_{i,j} \frac{\beta_i(x,t)}{\alpha_j(y,t)} dy = \max_{(x,t) \in \bar{G}} \int_{-kt}^x \varepsilon e^{-p(kt+x)} dy \leq \\ & \leq \max_{(x,t) \in \bar{G}} \left\{ \varepsilon e^{-p(kt+x)} (x+kt) \right\} = \varepsilon (l+2kT), \end{aligned}$$

$$\begin{aligned} \max_{(x,t) \in G} \int_{-kt}^x \max_{i,j} \frac{\beta_i(x,t)}{\beta_j(y,t)} dy &= \max_{(x,t) \in G} \int_{-kt}^x \frac{\mathcal{E} e^{-p(kt+x)}}{\mathcal{E} e^{-p(kt+y)}} dy = \\ &= \max_{(x,t) \in G} \int_{-kt}^x e^{p(y-x)} dy = \max_{(x,t) \in G} \frac{1}{p} (1 - e^{-p(kt+x)}) \leq \frac{1}{p}. \end{aligned}$$

$$\max_{(x,t) \in G} \int_{-kt}^x \max\left\{ \max_{i,j} \frac{\beta_i(x,t)}{\alpha_j(y,t)}; \max_{i,j} \frac{\beta_i(x,t)}{\beta_j(y,t)} \right\} dy \leq \varepsilon(l + 2kT) + \frac{1}{p}.$$

$$\begin{aligned} \rho(A[w^1], A[w^2]) &\leq (Le^{-pl} + \frac{L}{a} \max\left\{ \max_{i,j,y,\tau} \frac{\alpha_i(x,t)}{\alpha_j(y,\tau)}, \max_{i,j,y,\tau} \frac{\alpha_i(x,t)}{\beta_j(y,\tau)} \right\} + \\ &+ L\varepsilon + L(\varepsilon(l + 2kT) + \frac{1}{p})) \rho(w^1, w^2). \end{aligned}$$

$$p^*, \varepsilon^*$$

$$Le^{-pl} + L\varepsilon + L(\varepsilon(l + 2kT) + \frac{1}{p}) < \frac{1}{2},$$

$$\alpha_i^*, \beta_i^*, \quad \alpha_i, \beta_i \quad p = p^*, \varepsilon = \varepsilon^*.$$

$$M = \max_{(x,t) \in G} \left\{ \max_{i,j,y,\tau} \frac{\alpha_i^*(x,t)}{\alpha_j^*(y,\tau)}, \max_{i,j,y,\tau} \frac{\alpha_i^*(x,t)}{\beta_j^*(y,\tau)} \right\}.$$

$$a^*$$

$$p^* \leq a^* \mu, \quad p^* l - a^* \mu \leq -2p^* kT, \quad \frac{LM}{a^*} < \frac{1}{2}.$$

$$Q^* = Q$$

$$\alpha_i = \alpha_i^*, \beta_i = \beta_i^*$$

$$a = a^*.$$

$$A \quad Q^*.$$

(5)-(7).

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