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dmytro_bodnar@hotmail.com, maria.bubnyak@gmail.com

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$$1 + D \frac{a_n}{b_n} = 1 + \frac{a_1}{b_1 + \frac{a_2}{b_2 + \dots}}, \quad (1)$$

$a_n, b_n (n \geq 1)$ ó , k - , $a_{kn+p} = a_p$,
 $b_{kn+p} = b_p (n = 0, 1, \dots; p = \overline{1, k})$.

XIX . E. Galios, T. Thiele, A. Pringsheim. O. Perron [12],
 H. Wall [13], W. Jones, W. Thron [7].
 1966 . ()

$$1 + D \sum_{k=1}^{\infty} \frac{a_{i(k)}}{b_{i(k)}} = 1 + \sum_{i_1=1}^N \frac{a_{i(1)}}{b_{i(1)} + \sum_{i_2=1}^N \frac{a_{i(2)}}{b_{i(2)} + \dots}}, \quad (2)$$

$a_{i(k)}, b_{i(k)}$ ó , $i(k) = i_1, i_2, \dots, i_k$ ó , $1 \leq i_k \leq N, k \geq 1, N$ ó
 [10].

(2) [6], [9], [8] [5],

$$1 + D \sum_{k=1}^{\infty} \frac{a_{i(k)}}{1} = 1 + \sum_{i_1=1}^{i_0} \frac{a_{i(1)}}{1 + \sum_{i_2=1}^{i_1} \frac{a_{i(2)}}{1 + \dots}}, \quad (3)$$

$a_{i(k)}$ ó , $i(k) = i_1, i_2, \dots, i_k, 1 \leq i_k \leq i_{k-1}, k \geq 1, i_0 = N$.
 (3) \emptyset [2], [3,4].

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$$\left(1 + D \sum_{k=1}^{\infty} \frac{c_{i_k}}{1}\right)^{-1}, \quad (4)$$

$c_{i_k} \neq 0$ ó , $1 \leq i_k \leq i_{k-1}$, $k \geq 1$, $i_0 = N$.

(3)

1.

$a_{i(k)}$

(3)

$$P_{i(k),\varepsilon}(\gamma) = P_{i_k,\varepsilon}(\gamma) = \left\{ \omega \in C : |\omega| - \operatorname{Re}(\omega e^{-i\gamma}) \leq 2p_{i_k} (1 - \varepsilon) \cos^2 \gamma/2 \right\},$$

$-\pi < \gamma < \pi$, ε ó , ($0 < \varepsilon < 1$), $1 \leq i_k \leq i_{k-1}$, $i_0 = N$

$$p_{i_k} = \frac{1}{2N} \left(1 - \frac{i_k}{2N} \right). \quad (5)$$

1)

2)

(3)

a)

k ,

$$a_{i(k)} = 0 \quad (i_p = \overline{1, i_{p-1}}, p = \overline{1, k}),$$

b)

$$\sum_{k=1}^{\infty} \min \left(|a_{i(k)}|^{-1}; i_p = \overline{1, i_{p-1}}, p = \overline{1, k} \right), \quad i(k),$$

$a_{i(k)} = 0$

3)

$$K(\gamma) = \left\{ z \in C : \left| z - \frac{e^{-i\gamma/2}}{\cos \gamma/2} \right| \leq \frac{1}{\cos \gamma/2} \right\}. \quad (6)$$

3.23 [6, .114]

(2).

$$V_{i(k)} = V = \left\{ \omega \in C : \operatorname{Re}(\omega e^{-i\gamma/2}) \geq -\frac{1}{2N} \cos \gamma/2 \right\}$$

$$E_{i(k)} = E_{i_k} = \left\{ \omega \in C : |\omega| - \operatorname{Re}(\omega e^{-i\gamma}) \leq 2p_{i_k} \cos^2 \gamma/2 \right\},$$

$1 \leq i_k \leq i_{k-1}$, $i_0 = N$, $k \geq 1$, $-\pi < \gamma < \pi$, p_{i_k}

(5).

(4)

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2.

(4)

$c_{i_k} \in P_{i_k}(\gamma)$,

$$P_{i_k}(\gamma) = \left\{ \omega \in C : |\omega| - \operatorname{Re}(\omega e^{-i\gamma}) \leq 2p_{i_k} \cos^2 \gamma/2 \right\},$$

$-\pi < \gamma < \pi$, $i_k = \overline{1, N}$, p_{i_k} (5).

1)

(4)

;

2)

(4)

$K(\gamma)$,

(6).

$$\left(1 + D \sum_{k=1}^{\infty} \frac{z_{i_k}}{1} \right)^{-1}. \quad (7)$$

$$z_{i_k} \in P_{i_k}(\gamma) \quad (i_k = \overline{1, N}). \quad (7)$$

$$\sum_{k=1}^{\infty} \min(|z_i|^{-1}; i = \overline{1, N}) \delta$$

(7) ó

$$P_{i_k}(\gamma) \quad (i_k = \overline{1, N}),$$

$$\{F_n(z)\}_{n=1}^{\infty} \quad (7)$$

[6, .66],

$$\Delta = P_{i_k, \varepsilon}(\gamma).$$

$$z_{i_k} = c_{i_k}, \quad (4)$$

(4)

$$K(\gamma)$$

1.

I. $a \neq 0$ ó

$$a \in E(p),$$

$$E(p) = \{z \in \mathbb{C} : |\arg(z+p)| < \pi\}, \quad p \text{ ó}$$

$$P(\gamma) = \{\omega \in \mathbb{C} : |\omega| - \operatorname{Re}(\omega e^{-i\gamma}) \leq 2p \cos^2 \gamma/2\}, \quad -\pi < \gamma < \pi. \quad (8)$$

$$1) \quad \arg a = \pi, \quad a \in P(\gamma) \quad - \quad \gamma;$$

$$2) \quad \arg a \neq \pi, \quad a \in P(\gamma), \quad \gamma_1 \leq \gamma \leq \gamma_2,$$

$$\gamma_1 = \begin{cases} \arccos\left(\frac{A+B}{C}\right), & \cos \alpha \leq 1 - \frac{2p}{|a|} \\ -\arccos\left(\frac{A+B}{C}\right), & \cos \alpha > 1 - \frac{2p}{|a|} \end{cases}, \quad (9)$$

$$\gamma_2 = \arccos\left(\frac{A-B}{C}\right) \quad (10)$$

$$\alpha = \arg a, \quad A = (|a| - p)(p + |a| \cos \alpha), \quad B = |a| \sin \alpha \sqrt{2|a|p(1 + \cos \alpha)}, \quad C = p^2 + |a|^2 + 2|a|p \cos \alpha.$$

$$a = |a|e^{i\alpha}, \quad a \in P(\gamma),$$

$$|a| - \operatorname{Re}(ae^{i(\alpha-\gamma)}) \leq p(1 + \cos \gamma).$$

$$(p + |a| \cos \alpha) \cos \gamma + |a| \sin \alpha \sin \gamma \leq |a| - p.$$

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$\gamma,$

:

$$\left| \gamma - \arccos \frac{p + |a| \cos \alpha}{\sqrt{C}} \right| \leq \arccos \frac{|a| - p}{\sqrt{C}},$$

$$\gamma \in [\gamma_1, \gamma_2], \quad \gamma_1, \gamma_2 \quad (9)-$$

(10).

$$-\pi < \gamma_1 \leq \gamma_2 < \pi.$$

$$G_1 = \{\omega \in \mathbb{C} : |\arg(\omega + p)| < \pi\} \quad (11)$$

$$c_1 \in G_1(\gamma) \text{ ó}$$

$$P_k(\gamma) = \{\omega \in \mathbb{C} : |\omega| - \operatorname{Re}(\omega e^{-i\gamma}) \leq 2p_k \cos^2 \gamma/2\},$$

$$p_k = \frac{1}{2N} \left(1 - \frac{k}{2N}\right), \quad -\pi < \gamma < \pi, \quad k = \overline{1, N}.$$

G_s

$$c_s \quad (s = \overline{2, N})$$

$$(4) \quad , \quad c_r \quad (r = \overline{1, s-1})$$

$$1 \quad : \quad c_1 \in P_1(\gamma)$$

(9)-

(10)

$$, \quad \gamma \in [\gamma_1^1, \gamma_2^1], \quad \gamma_1^1 = \gamma_1 \quad \gamma_2^1 = \gamma_2,$$

$$a = c_1 \quad p = p_1.$$

(13), $G_k \neq \emptyset$ $[\Gamma_1^N, \Gamma_2^N] \neq \emptyset$. ,
 $\gamma \in [\Gamma_1^N, \Gamma_2^N]$: $c_k \in P_k(\gamma)$ ($k = \overline{1, N}$). , 2
 (4) $K(\gamma)$.
 $c_k \in P_k(\gamma)$, $\Gamma_1^N \leq \gamma \leq \Gamma_2^N$,
 $K(\Gamma_1^N) \cup K(\Gamma_2^N)$, $K(\gamma)$ (6).

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