[illegible] 517.524

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dmytro_bodnar@hotmail.com, maria.bubnyak@gmail.com
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[illegible]

$$1+\underset{n=1}{\overset{\infty}{D}}\frac{a_n}{b_n}=1+\cfrac{a_1}{b_1+\cfrac{a_2}{b_2+\ddots}}, \qquad \textbf{(1)}$$

[illegible] a_n, b_n $(n\geq 1)$ [illegible] k-[illegible] $a_{kn+p}=a_p$, $b_{kn+p}=b_p$ $(n=0,1,\ldots;\ p=\overline{1,k})$. [illegible] XIX [illegible] E. Galios, T. Thiele, A. Pringsheim. [illegible] O. Perron [12], H. Wall [13], W. Jones, W. Thron [7].

[illegible] 1966 [illegible] (1),

$$1+\underset{k=1}{\overset{\infty}{D}}\sum_{i_k=1}^{N}\frac{a_{i(k)}}{b_{i(k)}}=1+\sum_{i_1=1}^{N}\cfrac{a_{i(1)}}{b_{i(1)}+\sum\limits_{i_2=1}^{N}\cfrac{a_{i(2)}}{b_{i(2)}+\ddots}}, \qquad \textbf{(2)}$$

[illegible] $a_{i(k)}$, $b_{i(k)}$ [illegible] $i(k)=i_1,i_2,\ldots,i_k$ [illegible] $1\leq i_k\leq N$, $k\geq 1$, N [illegible] [10].

[illegible] (2) [illegible] [5], [illegible] [6], [illegible] [9], [illegible] [8] [illegible].

[illegible]

$$1+\underset{k=1}{\overset{\infty}{D}}\sum_{i_k=1}^{i_{k-1}}\frac{a_{i(k)}}{1}=1+\sum_{i_1=1}^{i_0}\cfrac{a_{i(1)}}{1+\sum\limits_{i_2=1}^{i_1}\cfrac{a_{i(2)}}{1+\ddots}}, \qquad \textbf{(3)}$$

[illegible] $a_{i(k)}$ [illegible] $i(k)=i_1,i_2,\ldots,i_k$, $1\leq i_k\leq i_{k-1}$, $k\geq 1$, $i_0=N$.

[illegible] (3) [illegible] [2], [illegible] [3,4]. [illegible]

[illegible] 1-[illegible]

$$\left(1+\underset{k=1}{\overset{\infty}{D}}\sum_{i_k=1}^{i_{k-1}}\frac{c_{i_k}}{1}\right)^{-1}, \qquad \textbf{(4)}$$

[illegible] $c_{i_k} \neq 0$ [illegible] $1 \le i_k \le i_{k-1}$, $k \ge 1$, $i_0 = N$.

[illegible]

[illegible] (3) [illegible].

[illegible] 1. [illegible] $a_{i(k)}$ [illegible] (3) [illegible]

$$P_{i(k),\varepsilon}(\gamma) = P_{i_k,\varepsilon}(\gamma) = \left\{\omega \in C : |\omega| - \operatorname{Re}\left(\omega e^{-i\gamma}\right) \le 2p_{i_k}(1-\varepsilon)\cos^2 \gamma/2\right\},$$

[illegible] $-\pi < \gamma < \pi$, ε [illegible] ($0 < \varepsilon < 1$), $1 \le i_k \le i_{k-1}$, $i_0 = N$ [illegible]

$$p_{i_k} = \frac{1}{2N}\left(1 - \frac{i_k}{2N}\right). \qquad \textbf{(5)}$$

[illegible]

1) [illegible];

2) [illegible] (3) [illegible]:

a) [illegible] k, [illegible] $a_{i(k)} = 0$ ($i_p = \overline{1, i_{p-1}}$, $p = \overline{1,k}$),

b) [illegible] $\sum_{k=1}^{\infty} \min\left(\left|a_{i(k)}\right|^{-1}; i_p = \overline{1, i_{p-1}}, p = \overline{1,k}\right)$ [illegible] $i(k)$, [illegible] $a_{i(k)} = 0$ [illegible];

3) [illegible] (3) [illegible]:

$$K(\gamma) = \left\{ z \in C : \left| z - \frac{e^{-i\gamma/2}}{\cos \gamma/2} \right| \le \frac{1}{\cos \gamma/2} \right\}. \qquad \textbf{(6)}$$

[illegible] 3.23 [6, [illegible]114] [illegible] (2). [illegible]

$$V_{i(k)} = V = \left\{\omega \in C : \operatorname{Re}\left(\omega e^{-i\gamma/2}\right) \ge -\frac{1}{2N}\cos \gamma/2\right\}$$

[illegible]

$$E_{i(k)} = E_{i_k} = \left\{\omega \in C : |\omega| - \operatorname{Re}\left(\omega e^{-i\gamma}\right) \le 2p_{i_k}\cos^2 \gamma/2\right\},$$

[illegible] $1 \le i_k \le i_{k-1}$, $i_0 = N$, $k \ge 1$, $-\pi < \gamma < \pi$, p_{i_k} [illegible] (5).

[illegible] (4) [illegible] 1 [illegible].

[illegible] 2. [illegible] (4) [illegible] $c_{i_k} \in P_{i_k}(\gamma)$, [illegible]

$$P_{i_k}(\gamma) = \left\{\omega \in C : |\omega| - \operatorname{Re}\left(\omega e^{-i\gamma}\right) \le 2p_{i_k}\cos^2 \gamma/2\right\},$$

$-\pi < \gamma < \pi$, $i_k = \overline{1, N}$, p_{i_k} [illegible] (5).

[illegible]

1) [illegible] (4) [illegible];

2) [illegible] (4) [illegible] $K(\gamma)$, [illegible] (6).

[illegible]

$$\left(1+\underset{k=1}{\overset{\infty}{D}}\sum_{i_k=1}^{i_{k-1}}\frac{z_{i_k}}{1}\right)^{-1}. \qquad \textbf{(7)}$$

[illegible] $z_{i_k} \in P_{i_k}(\gamma)$ $(i_k = \overline{1,N})$. [illegible] 1, [illegible] $\sum_{k=1}^{\infty} \min\left(|z_i|^{-1}; i = \overline{1,N}\right)$ – [illegible].

[illegible] (7) – [illegible] $P_{i_k}(\gamma)$ $(i_k = \overline{1,N})$, [illegible] $\{F_n(z)\}_{n=1}^{\infty}$ [illegible] (7) [illegible] [6, [illegible] 66], [illegible] $\Delta = P_{i_k,\varepsilon}(\gamma)$. [illegible] $z_{i_k} = c_{i_k}$, [illegible] (4) [illegible] $K(\gamma)$ [illegible] 1.

[illegible] 1. [illegible] $a \neq 0$ – [illegible], [illegible] $a \in E(p)$, [illegible] $E(p) = \{z \in C : |\arg(z+p)| < \pi\}$, p – [illegible],

$$P(\gamma) = \left\{\omega \in C : |\omega| - \operatorname{Re}\left(\omega e^{-i\gamma}\right) \le 2p\cos^2 \tfrac{\gamma}{2}\right\}, \ -\pi < \gamma < \pi . \qquad \textbf{(8)}$$

[illegible]

1) [illegible] $\arg a = \pi$, [illegible] $a \in P(\gamma)$ [illegible] γ;

2) [illegible] $\arg a \neq \pi$, [illegible] $a \in P(\gamma)$, $\gamma_1 \le \gamma \le \gamma_2$, [illegible]

$$\gamma_1 = \begin{cases} \arccos\left(\dfrac{A+B}{C}\right), & [illegible] \cos\alpha \le 1 - \dfrac{2p}{|a|} \\ -\arccos\left(\dfrac{A+B}{C}\right), & [illegible] \cos\alpha > 1 - \dfrac{2p}{|a|} \end{cases}, \qquad \textbf{(9)}$$

$$\gamma_2 = \arccos\left(\frac{A-B}{C}\right) \qquad \textbf{(10)}$$

[illegible] $\alpha = \arg a$, $A = (|a| - p)(p + |a|\cos\alpha)$, $B = |a||\sin\alpha|\sqrt{2|a|p(1+\cos\alpha)}$, $C = p^2 + |a|^2 + 2|a|p\cos\alpha$.

[illegible] $a = |a|e^{i\alpha}$. [illegible] $a \in P(\gamma)$, [illegible]

$$|a| - \operatorname{Re}\left(ae^{i(\alpha-\gamma)}\right) \le p(1+\cos\gamma).$$

[illegible]

$$(p + |a|\cos\alpha)\cos\gamma + |a|\sin\alpha\sin\gamma \le |a| - p .$$

[illegible] γ, [illegible]:

$$\left|\gamma - \arccos\frac{p + |a|\cos\alpha}{\sqrt{C}}\right| \le \arccos\frac{|a| - p}{\sqrt{C}},$$

[illegible] $\gamma \in [\gamma_1, \gamma_2]$, [illegible] γ_1, γ_2 [illegible] (9)-(10).

[illegible] $-\pi < \gamma_1 \le \gamma_2 < \pi$. [illegible]

[illegible]

$$G_1 = \{\omega \in C : |\arg(\omega + p)| < \pi\} \qquad \textbf{(11)}$$

[illegible] $c_1 \in G_1(\gamma)$ – [illegible].

[illegible]

$$P_k(\gamma) = \left\{\omega \in C : |\omega| - \operatorname{Re}\left(\omega e^{-i\gamma}\right) \le 2p_k\cos^2 \tfrac{\gamma}{2}\right\},$$

[illegible] $p_k = \frac{1}{2N}\left(1 - \frac{k}{2N}\right)$, $-\pi < \gamma < \pi$, $k = \overline{1,N}$.

[illegible] G_s [illegible] c_s $\left(s = \overline{2,N}\right)$ [illegible] (4) [illegible] c_r $\left(r = \overline{1, s-1}\right)$ [illegible].

[illegible] 1 [illegible]: $c_1 \in P_1(\gamma)$ [illegible] $\gamma \in [\gamma_1^1, \gamma_2^1]$, [illegible] $\gamma_1^1 = \gamma_1$ [illegible] $\gamma_2^1 = \gamma_2$, [illegible] (9)-(10) [illegible] $a = c_1$ [illegible] $p = p_1$.

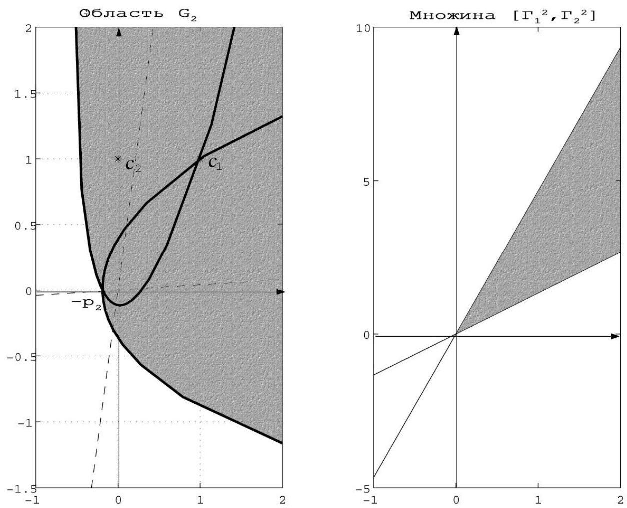
[illegible]

$$G_2 = G_2(c_1) = \bigcup_{\gamma \in [\gamma_1^1, \gamma_2^1]} P_2(\gamma).$$

[illegible] $c_2 \in G_2$.

[illegible] c_3. [illegible] $c_2 \in P_2(\gamma)$, [illegible] $\gamma \in [\gamma_1^2, \gamma_2^2]$, [illegible] $\gamma_1^2 = \gamma_1$ [illegible] $\gamma_2^2 = \gamma_2$. [illegible] γ_1 [illegible] γ_2 [illegible] (9)-(10), [illegible] $a = c_2$ [illegible] $p = p_2$. [illegible] $[\Gamma_1^2, \Gamma_2^2] = [\gamma_1^1, \gamma_2^1] \cap [\gamma_1^2, \gamma_2^2]$ [illegible]

$$G_3 = G_3(c_1, c_2) = \bigcup_{\gamma \in [\Gamma_1^2, \Gamma_2^2]} P_3(\gamma).$$



[illegible] 1

[illegible] 1 [illegible] G_2 [illegible] c_2, [illegible] $c_1 = 1 + i$. [illegible] $c_2 = i$. [illegible] $[\Gamma_1^2, \Gamma_2^2]$ [illegible] G_3.

[illegible] (4), [illegible].

[illegible] G_1, G_2,..., G_{s-1} [illegible] c_1, c_2,..., c_{s-1} [illegible].

[illegible]

$$[\Gamma_1^{s-1}, \Gamma_2^{s-1}] = \bigcap_{j=1}^{s-1} [\gamma_1^j, \gamma_2^j] \qquad \textbf{(12)}$$

[illegible]

$$G_s = G_s(c_1, c_2, \ldots, c_{s-1}) = \bigcup_{\gamma \in [\Gamma_1^{s-1}, \Gamma_2^{s-1}]} P_s(\gamma) \; \mathbf{(}s = \overline{2, N}\mathbf{).} \qquad \textbf{(13)}$$

[illegible] G_s [illegible] c_s [illegible] $c_s \in P_s(\gamma)$ [illegible] $\Gamma_1^N \le \gamma \le \Gamma_2^N$, $s = \overline{1, N}$.

[illegible] 3. [illegible] (4) [illegible]: $c_s \in G_s \; (s = \overline{1, N})$, [illegible] G_1 [illegible] (11), [illegible] $G_s(c_1, c_2, \ldots, c_{s-1})$ [illegible] (12)- (13) $s = \overline{2, N}$.

[illegible]

1) [illegible] (4) [illegible];

2) [illegible] $K(\Gamma_1^N) \cup K(\Gamma_2^N)$, [illegible] $K(\gamma)$ [illegible] (6), [illegible] Γ_1^N, Γ_2^N [illegible] (12).

[illegible] G_k [illegible] (13), [illegible] $G_k \neq \varnothing$ [illegible] $\left[\Gamma_1^N, \Gamma_2^N\right] \neq \varnothing$. [illegible] $\gamma \in \left[\Gamma_1^N, \Gamma_2^N\right]$ [illegible]: $c_k \in P_k(\gamma)$ ($k = \overline{1, N}$). [illegible] 2 [illegible] (4) [illegible] $K(\gamma)$ [illegible].

[illegible] $c_k \in P_k(\gamma)$, [illegible] $\Gamma_1^N \le \gamma \le \Gamma_2^N$, [illegible] $K\left(\Gamma_1^N\right) \cup K\left(\Gamma_2^N\right)$, [illegible] $K(\gamma)$ [illegible] (6).

[illegible]

[illegible] ([illegible]). [illegible] 1-[illegible]. [illegible] 1-[illegible].

[illegible]

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