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$\alpha \in (0,1)$  ([1] - [13]).

$\{V_0, V_1\}$

$A$

$V_0$

$V_1 \subset V_0$ .

1.

$$D_t^\alpha g(t) = f_{-\alpha}(t) * g(t) \quad \alpha \in R$$

$g(t)$ ,

$$f_\lambda(t) = \frac{\theta(t)t^{\lambda-1}}{\Gamma(\lambda)} \quad \lambda > 0 \quad f_\lambda(t) = f'_{1+\lambda}(t) \quad \lambda \leq 0,$$

$$\Theta(t) = \dots, \Gamma(\lambda) = \dots \quad ([14], \text{ c. } 87),$$

$$I_t^\alpha g(t) = f_\alpha(t) * g(t) = \frac{\theta(t)}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} g(s) ds, \quad \alpha > 0,$$

$$\partial_t^\alpha g := \frac{d}{dt}(f_{1-\alpha} * g)(t) - f_{1-\alpha}(t)g(0), \quad \alpha > 0 \quad g(t).$$

$$(f_\lambda * f_\mu)(t) = f_{\lambda+\mu}(t), \quad \lambda, \mu \in R \quad ([14], \text{ c. } 87).$$

$$D_t^\alpha g(t) = \frac{d}{dt} I_t^{1-\alpha} g(t), \quad \partial_t^\alpha g(t) = D_t^\alpha (g(t) - g(0)),$$

$$I_t^\alpha \partial_t^\alpha g(t) = g(t) - g(0), \quad \partial_t^\alpha I_t^\alpha g(t) = g(t), \quad t > 0, \quad \alpha > 0.$$

$A$  ó

$(V_0, \|\cdot\|_{V_0})$

$V_1 \subset V_0$ .

$V_1$

$A$ ,

$V_1$

$$E_{10}: V_1 \rightarrow V_0$$

[6] (Prop.1)

$A$  ó

$\{\Phi_A(t)\}_{t \geq 0}$

$V_0$ ,

$$R(\lambda - A) = (\lambda - A)^{-1}$$

$$V_0 \quad [15] \quad \int_0^\infty e^{-\lambda t} \Phi_A(t) h dt = (\lambda - A)^{-1} h \quad \operatorname{Re} \lambda > \omega(\Phi_A),$$

$$\omega(\Phi_A) = \inf \left\{ \omega > 0 : \sup_{t \geq 0} \|e^{-\omega t} \Phi_A(t)\|_{\mathcal{L}(V_0)} < \infty \right\},$$

$v \in C([0, \infty); V_1)$

$\alpha \in (0,1), h \in V_1$  ó

$$D_t^\alpha v(t) = Av(t) + f_{1-\alpha}(t)v(0), \quad v(0) = h, \quad (1)$$

$t \geq 0$

$\partial_t^\alpha v$ ,

$L[\partial_t^\alpha v]$ ,

$v(t)$

$$I_t^\alpha v(t) \in V_1 \quad v(t) \\ \partial_t^\alpha v(t) = Av(t), \quad v(0) = h, \tag{2}$$

$$v(t) = AI_t^\alpha v(t) + h, \quad t \geq 0. \tag{3}$$

$$v \in C([0, \infty); V_1), \quad I_t^\alpha v(t) \in V_1, \quad v(t) \tag{13}$$

(1), (2). [6] (Prop.1)  $v'(t) \in V_0$   $v(t)$

$$S_{\alpha,A}(t)h = \int_0^\infty \frac{t}{\alpha s^{1+\frac{1}{\alpha}}} g_\alpha\left(\frac{t}{s^\alpha}\right) \Phi_A(s) h ds = \int_0^\infty \Phi_A\left(\left(\frac{t}{s}\right)^\alpha\right) g_\alpha(s) h ds,$$

$$g_\alpha \text{ ó } \exp(-\lambda^\alpha), \quad \int_0^\infty e^{-\lambda t} g_\alpha(t) dt = \exp(-\lambda^\alpha).$$

[5] [15] , A

$$\{S_{\alpha,A}(t)\}_{t \geq 0} \quad V_0,$$

$$Z\left(\frac{\pi}{2} + \alpha'\right) = \left\{ re^{i\omega}; r > 0, |\omega| < \frac{\pi}{2} + \alpha' \right\}, \quad \alpha' < \alpha, \quad t,$$

$$\{f_{1-\alpha}(t) * S_{\alpha,A}(t)\}_{t \geq 0} \quad Z(\alpha), \quad v(t) = S_{\alpha,A}(t)h, \quad t > 0$$

(1)  $h \in V_0$  ,  $h$ ,  
 $g \in L_1([0, T], V_0)$   $T > 0$  ,

$$u(t) = AI_t^\alpha u(t) + h + \int_0^t g(s) ds, \quad t \in [0, T]$$

$$u(t) = S_{\alpha,A}(t)h + \int_0^t S_{\alpha,A}(t-s)g(s) ds, \quad t \in [0, T].$$

[10]  $C', C_m, m \in N,$

$$\|S_{\alpha,A}(t)\|_{L(V_0)} \leq \frac{C'}{1 + |r(A)|t^\alpha}, \quad t \geq 0, \tag{4}$$

$$\left\| \left(\frac{d}{dt}\right)^m S_{\alpha,A}(t) \right\|_{\mathcal{L}(V_0)} \leq \frac{C_m}{t^m}, \quad t > 0, \quad m \in N.$$

(1)

$$[8], \quad D_t^\alpha u(t) = Au(t) + D_t^{\alpha-1}g(t) \quad \alpha \in (1, 2)$$

[11], ,

[15], [16].

$$l_\omega := \{re^{i\omega} : r > 0\} \text{ ó } \omega \in [0, 2\pi],$$

$$\omega_0 \in \left(\frac{\pi}{2}, \pi\right) \quad C$$

$$\{0\} \quad \Lambda_0 := \bigcup \{l_\omega : \omega \in [-\omega_0, \omega_0]\} \quad \Lambda := \Lambda_0 \bigcup \{0\}.$$

$$A \quad A: V_1 \mapsto V_0,$$

$$(\lambda E_{10} - A)^{-1} \quad \lambda \in \Lambda$$

$$L(V_0; V_1) \quad V_0 \quad V_1, \dots$$

$$K(A) := \left\{ A \in L(V_1; V_0) : \sup_{\lambda \in \Lambda} (\lambda E_{10} - A)^{-1}_{L(V_0, V_1)} = K(A) < \infty \right\},$$

$K(A)$

$$\rho(A) := \left\{ \lambda : (\lambda E_{10} - A)^{-1} \in L(V_0; V_1) \right\} \quad \sigma(A) := \mathbf{C} \setminus \rho(A)$$

$A$  , [17] ( ) ,  $A$

$$\rho(A) \ni \lambda \mapsto (\lambda E_{10} - A)^{-1} \in L(V_0; V_1).$$

$$R(\lambda, A) := E_{10} (\lambda E_{10} - A)^{-1} \in L(V_0, V_0) := L(V_0) \quad \lambda \in \rho(A).$$

$$r(A) := \sup \{ \operatorname{Re} \lambda : \lambda \in \sigma(A) \} \quad V_0. \quad A$$

$A$

$$S_{\alpha, A}(t) = \frac{1}{2\pi i} \int_{\gamma} e^{\lambda t} \lambda^{\alpha-1} (\lambda^\alpha - A)^{-1} d\lambda, \quad t \geq 0$$

$$\gamma \in r(A) + \Lambda \quad [10].$$

$$A \text{ ó } \quad r(A) \quad \omega_0 \in \left( \frac{\pi}{2}, \pi \right).$$

$$J \in A. \quad [17]-[19], \quad -J:$$

$$(-J)^{-g} = \frac{1}{2\pi i} \int_{\Gamma_{a, \omega}} \frac{R(\lambda, J)}{(-\lambda)^g} d\lambda \in L(V_j), \quad 0 < g < 1, \quad j = 0, 1,$$

$$\Gamma_{a, \omega} := \{ re^{-i\omega} : r \geq a \} \cup \{ ae^{i\tau} : \tau \in [\omega, 2\pi - \omega] \} \cup \{ re^{i\omega} : r \geq a \}$$

$$a : 0 < a < r(A) \quad \omega : \omega_0 - c \leq \omega \leq \omega_0, \quad \frac{\pi}{2} < \omega_0 - c < \pi. \quad (-J)^{-g}$$

[19]

$$(-J)^{-g} (-J)^{-g'} = (-J)^{-g-g'}, \quad \forall g, g' > 0.$$

$V_g$

$$J^g \quad (0 < g < 1)$$

$$\|x\|_{V_g} := \|(-J)^g x\|_{V_0}. \quad V_g = [\cdot, \cdot]_g \text{ ó } \{V_0; V_1\},$$

([18], 1.15.3).

$$[19] \quad , \quad J_{V_1} \quad (A_{V_1})$$

$$\emptyset \quad \omega_0 \quad \{V_0; V_1\}, \quad [17],$$

c. 1706171,

$$\Phi_J(t) \quad V_0 \quad V_1$$

$V_0, V_1:$

$$\Phi_J(t) = \frac{1}{2\pi i} \int_{\Gamma_{a, \omega}} e^{(t)\lambda} R(\lambda, J) d\lambda \in L(V_0) \cap L(V_1), \quad t \geq 0$$

$$\Phi_A(t) \in L(V_0) \cap L(V_1), \quad J_{V_\theta} \quad A_{V_\theta}$$

$$\emptyset \quad \omega_0 \quad \{V_\theta; V_{1+\theta}\}.$$

2.

$$(A): A \in A, \quad \alpha, \theta \in (0, 1), \quad X \in L(V_\theta, V_0), \quad h \in V_1.$$

$$(G_{in}) : g \in C([0, T]; V_1), \quad f_{\alpha-1} * g \in C([0, T]; V_1).$$

$$D_t^\alpha u(t) = (A + X)u(t) + D_t^{\alpha-1} g(t) + f_{1-\alpha}(t)u(0), \quad u(0) = h. \quad (5)$$

$$C^\alpha := \{v \in C([0, T]; V_1) : \exists D^\alpha u \in C([0, T]; V_0)\}$$

$$u(t) \in C([0, T]; V_0), \quad u \in C([0, T]; V_1) \quad (5)$$

$$I^\alpha u = f_\alpha * u \in C([0, T]; V_1).$$

$$1. \quad A \in A, \quad \theta \in (0, 1), \quad X \in L(V_\theta, V_0), \quad A + X|_{V_1} \in A, \quad \Lambda \subset \rho(A) \cap \rho(A + X)$$

$$\|R(\lambda, A + X)\|_{L(V_0)} \leq 2 \|R(\lambda, A)\|_{L(V_0)}, \quad \forall \lambda \in \Lambda.$$

$$\left\| \left( \lambda E_{10} - \frac{A}{s} \right)^{-1} \right\|_{L(V_0, V_g)} \leq \frac{C'' s^\theta}{\lambda^{1-\theta}}, \quad \forall \lambda \in \Lambda \setminus \{0\}, \quad \forall s > 0.$$

$$\left\| (\lambda E_{10} - A)^{-1} \right\|_{L(V_0, V_g)} \leq \frac{C''}{a^{1-\theta}}, \quad a = \left[ 2C'' \|X\|_{L(V_g, V_1)} \right]^{\frac{1}{1-\theta}},$$

$$\|X(\lambda E_{10} - A)^{-1}\|_{L(V_0)} \leq \|X\|_{L(V_g, V_0)} \left\| (\lambda E_{10} - A)^{-1} \right\|_{L(V_g, V_0)} \leq \frac{1}{2}.$$

$$L(V_0)$$

$$\left[ E_{00} - XE_{1\theta}(\lambda E_{10} - A)^{-1} \right]^{-1} = \sum_{k=1}^{\infty} \left[ XE_{1\theta}(\lambda E_{10} - A)^{-1} \right]^k$$

$$\left\| \left[ E_{00} - XE_{1\theta}(\lambda E_{10} - A)^{-1} \right]^{-1} \right\|_{L(V_0)} \leq 2 \quad \lambda \in \Lambda.$$

$$(\lambda E_{10} - A - XE_{1\theta})^{-1} = (\lambda E_{10} - A)^{-1} \left[ E_{00} - XE_{1\theta}(\lambda E_{10} - A)^{-1} \right]^{-1}$$

$$\left\| (\lambda E_{10} - A - XE_{1\theta})^{-1} \right\|_{L(V_0, V_g)} \leq 2 \left\| (\lambda E_{10} - A)^{-1} \right\|_{L(V_0, V_1)}$$

$$\lambda \in \Lambda, \quad X \in L(V_g, V_0), \quad X \in L(U, V_0)$$

$$\|X\|_{L(U, V_0)}, \quad U \in \{V_0, V_1\}$$

[20].

$$2. \quad (A) \quad (G_{in}) \quad u \in C^\alpha, \quad (5)$$

$$u \in C([0, T]; V_1)$$

$$u(t) = (A + X)(f_\alpha(t) * u(t)) + \int_0^t g(s) ds + h, \quad t \in [0, T]. \quad (6)$$

$$1. \quad A + X|_{V_1} \in A, \quad \Lambda \subset \rho(A) \cap \rho(A + X), \quad g \in C([0, T]; V_1)$$

$$\int_0^t g(s) ds, \quad t \in [0, T], \quad C([0, T]; V_1).$$

$$I_t^\alpha u(t) \quad (5), \quad u(t) \text{ ó } \quad (5) \quad ($$

$$I_t^\alpha (A + X)u(t) = (A + X)I_t^\alpha u(t) = (A + X)(f_\alpha * u)(t) \quad , \quad , \quad u$$

$$u \in C([0, T]; V_1) \text{ ó } \quad (6), \quad D_t^\alpha$$

$$(6) \quad , \quad \int_0^t g(s)ds = f_1(t) * g(t)$$

$$D_t^\alpha \left( \int_0^t g(s)ds \right) = f_{-\alpha} * (f_1(t) * g(t)) = (f_{-\alpha} * f_1(t)) * g(t) = f_{1-\alpha} * g(t) = D_t^{\alpha-1} g(t)$$

$$D_t^\alpha u(t) \quad V_0 \quad ( \quad u \in C^\alpha ) \quad u \quad (5).$$

$$1. \quad (A), (G_{lin}) \quad , \quad u \in C^\alpha \quad (5),$$

$$u(t) = S_{\alpha, A+XE_{1,g}}(t)h + \int_0^t S_{\alpha, A+XE_{1,g}}(t-s)g(s)ds, \quad t \in [0, T] \quad (7)$$

$$E_{1,g} \text{ ó } \quad V_1 \rightarrow V_g.$$

$$1 \quad A + XE_{1,g} \in A \quad \Lambda \subset \rho(A) \cap \rho(A + X),$$

$$\Phi_{\alpha, A+XE_{1,g}}(t) \quad [20], [21], \quad S_{\alpha, A+XE_{1,g}}(t) \quad v_1(t) = S_{\alpha, A+XE_{1,g}}(t)h$$

$$C^\alpha, \quad (1),$$

$$\frac{d}{dt} I_t^{1-\alpha} S_{\alpha, A+XE_{1,g}}(t) = (A + X)S_{\alpha, A+XE_{1,g}}(t) + f_{1-\alpha}(t) \quad V_1, \quad t \in [0, T].$$

$$\Phi_{\alpha, A+XE_{1,g}}(t), \quad S_{\alpha, A+XE_{1,g}}(t), \quad (G_{lin})$$

$$, \quad v_2(t) = \int_0^t S_{\alpha, A+XE_{1,g}}(t-s)g(s)ds \quad C^\alpha.$$

$$A + XE_{1\theta},$$

$$g \in C([0, T]; V_1), \quad , \quad v_2(t) \quad (5) \quad h = 0.$$

$$\begin{aligned} D^\alpha (v(t) + v_2(t)) &= D^\alpha \left[ S_{\alpha, A+XE_{1\theta}}(t)h + \int_0^t S_{\alpha, A+XE_{1\theta}}(t-s)g(s)ds \right] = \\ &= (A + X)S_{\alpha, A+XE_{1,g}}(t) + f_{1-\alpha}(t)h + \frac{d}{dt} f_{1-\alpha}(t) * (S_{\alpha, A+XE_{1\theta}}(t) * g(t)) = \\ &= (A + X)S_{\alpha, A+XE_{1,g}}(t) + f_{1-\alpha}(t)h + \left( \frac{d}{dt} f_{1-\alpha}(t) * S_{\alpha, A+XE_{1,g}}(t) \right) * g(t) = \\ &= (A + X)S_{\alpha, A+XE_{1,g}}(t) + f_{1-\alpha}(t)h + \left[ (A + X)S_{\alpha, A+XE_{1,g}}(t) + f_{1-\alpha}(t) \right] * g(t) = \\ &= (A + X)S_{\alpha, A+XE_{1,g}}(t) + f_{1-\alpha}(t)h + (A + X)(S_{\alpha, A+XE_{1,g}}(t) * g(t)) + f_{1-\alpha}(t) * g(t) = \\ &= (A + X) \left[ S_{\alpha, A+X}(t) + S_{\alpha, A+XE_{1,g}}(t) * g(t) \right] + f_{1-\alpha}(t)h + f_{1-\alpha}(t) * g(t) = \\ &= (A + X) \left[ S_{\alpha, A+XE_{1,g}}(t) + \int_0^t S_{\alpha, A+XE_{1,g}}(t-s)g(s)ds \right] + f_{1-\alpha}(t)h + D_t^{\alpha-1} g(t). \end{aligned}$$

$$, \quad (7) \quad C^\alpha \quad (5).$$

$$u_1, u_2 \text{ ó } \quad (5), \quad u = u_1 - u_2$$

$$D_t^\alpha u(t) = (A + X)u(t), \quad u(0) = 0,$$

$$u_1(t) = u_2(t), t \in [0, T].$$

$$C([0, T]; V_1) \quad (A), (G_{in}). \quad (7). \quad (6)$$

3.

A,

$$L(x, D) = \sum_{|\gamma| \leq 2m} a_\gamma(x) D^\gamma, \quad a_\gamma(x) \in L_\infty(\Omega)$$

$$\text{Re} a(x, \zeta) = \text{Re} \sum_{|\gamma|=2m} a_\gamma(x) \zeta^\gamma > 0, \quad \zeta \in \mathbf{R}^n \setminus \{0\}, \quad x \in \Omega,$$

$$\gamma = (\gamma_1, \dots, \gamma_n) \quad , \quad |\gamma| = \gamma_1 + \dots + \gamma_n, \quad D^\gamma = \frac{\partial^{|\gamma|}}{\partial^{\gamma_1} \dots \partial^{\gamma_n}}, \quad \zeta^\gamma = \zeta_1^{\gamma_1} \dots \zeta_n^{\gamma_n},$$

$$a_\gamma \in L_\infty(\bar{\Omega}), \quad |\gamma| = 2m \quad \bar{\Omega} \subset \mathbf{R}^n \quad C^\infty,$$

$$\{B_j(x, D)\}_{j=1}^m = \left\{ \sum_{|\gamma| \leq k_j} b_{j,\gamma}(x) D^\gamma \right\}_{j=1}^m \quad ([22], \text{ c. } 178), \quad L(x, D)$$

$$[23] \quad k_j < 2m\vartheta - \frac{1}{p} \quad j = 1, \dots, m,$$

$$D^\alpha u_t = [L(x, D) + L_1(x, \theta)] u + D^{\alpha-1} f(x, t) + f_{1-\alpha}(t) h(x) \quad x \in \Omega, t \in [0, T] \quad B_j u|_{\partial\Omega} = 0, \\ j = 1, \dots, m \quad u(x, 0) = h(x) \quad (8)$$

$$C^\alpha(\Omega \times [0, T]; H_{p, \{B_j\}}^{2m}(\Omega)) := \left\{ v \in C(\Omega \times [0, T]; H_{p, \{B_j\}}^{2m}(\Omega)); \right.$$

$$\left. \exists D_t^\alpha v \in C(\Omega \times (0, T]; H_{p, \{B_j\}}^{2m}(\Omega)) \right\}$$

$$L_1(x, \theta) \quad \acute{o}$$

$$a_1(x, \xi) = \left[ \sum_{|\alpha| \leq 2m} a_{1\alpha}(x) (-i\xi)^\alpha \right]^\vartheta,$$

$$a_{1\alpha} \quad \acute{o} \quad \Omega,$$

$$L_1(x, \theta)v(x) = F_{\xi \rightarrow x}^{-1} \left[ a_1(x, \xi) (F_{x \rightarrow \xi}[v])(\xi) \right], \quad v \in H_{p, \{B_j\}}^{2m}(\Omega),$$

$$H_p^{2m}(\Omega) \quad \acute{o} \quad ([24], \text{ c. } 79),$$

$$V_1 = H_{p, \{B_j\}}^{2m}(\Omega) := \left\{ v \in H_p^{2m}(\Omega) : B_j(x, D)v|_{\partial\Omega} = 0; j = \overline{1, m} \right\},$$

$$H_{p, \{B_j\}}^{2m}(\Omega) \quad H_p^{2m}(\Omega) \quad v(x) = 0$$

$x \notin \Omega$ .

$$(Av)(x) = L(x, D)v(x),$$

$$V_0 = L_p(\Omega) \quad (1 < p < \infty)$$

$$V_1,$$

$$V_\vartheta = H_{p, \{B_j\}}^{2m\vartheta}(\Omega) := \left\{ v \in H_p^{2m\vartheta}(\Omega) : B_j(x, D)v|_{\partial\Omega} = 0; j = \overline{1, m} \right\}$$

$$H_p^{2m\vartheta}(\Omega) \quad 2m\vartheta. \quad L_p(\Omega)$$

$$H_p^{2m\vartheta}(\Omega). \quad , \quad f \in C(\Omega \times 0, T; H_{p, \{B_j\}}^{2m}(\Omega)),$$

$$h \in C(\Omega; H_{p, \{B_j\}}^{2m}(\Omega)).$$

[8]

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