

519.6

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[1,2]:

$$F(x)u \equiv \iint_S \frac{(1-|y|)^\beta u(y)}{|x-y|} dS_y = f(x), \tag{1}$$

$$G(x)u \equiv f.p. \iint_S \frac{(1-|y|)^\alpha u(y)}{|x-y|^\beta} dS_y = g(x)$$

$x = \{x_1, x_2\}$ ,  $y = \{y_1, y_2\}$ ,  $S$  ó ,  $x \in S$ ,  $\beta > -1, \alpha \geq 0$ . f.p.

(1)

$F(x)u$

$G(x)u$ .

(1)

$u(y)$

$$y = \{\rho \cos \varphi, \rho \sin \varphi\}, \quad x = \{r \cos \phi, r \sin \phi\}$$

$$u(y) \equiv u(\rho, \varphi) \quad \varphi \in [0, 2\pi], \quad \tau_l = 2\pi l/m, \quad l = \overline{0, m-1}$$

$$\rho \in [0, p], \quad u(\rho, \tau_l) \quad \rho$$

$$\rho_k \in [0, p], \quad \{\rho_k\},$$

$$P_n^{(1,\beta)}(1-2\rho) \quad P_n^{(1,\alpha)}(1-2\rho).$$

(1)

$F(x)u$

$$F_{kj}(r, \phi; \tau_l) = \int_0^1 \int_0^{2\pi} \frac{\bar{P}_k^{(\beta)}(\rho) \cos[j(\varphi - \tau_l)] d\rho d\varphi}{(\rho^2 + r^2 - 2\rho r \cos(\varphi - \phi))^{1/2}}, \tag{2}$$

$G(x)u$  ó

$$G_{kj}(r, \phi; \tau_l) = f.p. \int_0^1 \int_0^{2\pi} \frac{\bar{P}_k^{(\alpha)}(\rho) \cos[j(\varphi - \tau_l)] d\rho d\varphi}{(\rho^2 + r^2 - 2\rho r \cos(\varphi - \phi))^{3/2}}, \tag{3}$$

$$\bar{P}_k^{(\gamma)}(\rho) = \rho(1-\rho)^\gamma P_k^{(1,\gamma)}(1-2\rho), \quad k = \overline{0, n-1}, \quad j = \overline{0, p}.$$

(2)

[3]

$$\int_0^{2\pi} \frac{\cos[j(\phi - \tau_l)] d\phi}{\sqrt{\rho^2 + r^2 - 2\rho r \cos(\phi - \phi)}} = \frac{(-1)^j 2\pi R^j(\rho, r)}{(1/2)_j \sqrt{|\rho^2 - r^2|}} \cos j(\phi - \tau_l).$$

$$R^j(\rho, r) = P_{-1/2}^j\left(\frac{\rho^2 + r^2}{|\rho^2 - r^2|}\right), \quad P_v^\mu(z) \text{ ó } \quad (a)_j \text{ ó}$$

$$P_v^\mu(z) \text{ ó } \quad P_v^\mu(z)$$

$$-\infty + 1.$$

$$|1-x|^v P_v^\mu\left(\frac{1+x}{|1-x|}\right) [4],$$

$$F_{kj}(r, \phi; \tau_l) = J(\phi, \tau) G_{66}^{33} \left( r^2 \mid \begin{matrix} a_1, \dots, a_6 \\ b_1, \dots, b_6 \end{matrix} \right), \quad (4)$$

$$a_1 = (1-k)/2, a_2 = -k/2, a_3 = (1-j)/2, a_4 = (1+j)/2, a_5 = (2+k+\beta)/2, a_6 = (3+k+\beta)/2,$$

$$b_1 = j/2, b_2 = 1/2, b_3 = 1, b_4 = -j/2, b_5 = 0, b_6 = 1/2, \quad G_{66}^{33} \left( r^2 \mid \begin{matrix} a_1, \dots, a_6 \\ b_1, \dots, b_6 \end{matrix} \right) \text{ ó } G \text{ ó}$$

$$J(\phi, \tau) = 2^{-\beta} \pi \Gamma(\beta + k + 1) \cos j(\phi - \tau) / k!. \quad (3) \quad [5]$$

$$f.p. \iint_s \frac{f(y)}{|x-y|^3} dS_y = \Delta_x \iint_s \frac{f(y)}{|x-y|} dS_y, \quad (5)$$

$\Delta_x$ -

$$G_{nj}(r, \phi; \tau) = J(\phi, \tau) \left( \begin{matrix} -j^2 G_{66}^{33} \left( r^2 \mid \begin{matrix} a_1, \dots, a_6 \\ b_1, \dots, b_6 \end{matrix} \right) + \\ + 4 G_{77}^{34} \left( r^2 \mid \begin{matrix} c_1, \dots, c_7 \\ d_1, \dots, d_7 \end{matrix} \right) \end{matrix} \right), \quad (6)$$

$$a_1 = (1-j)/2, a_2 = -n/2, a_3 = (1-n)/2, a_4 = (j+1)/2, a_5 = (2+n+\alpha)/2, a_6 = a_5 + 1/2;$$

$$b_1 = 1/2, b_2 = 1, b_3 = j/2, b_4 = 0, b_5 = b_1, b_6 = -b_3; c_1 = -1, c_{i+1} = a_i - 1, i = \overline{1, 6};$$

$$d_1 = d_4 = -b_1, d_2 = d_5 = d_6 = 0, d_3 = b_3 - 1, d_7 = b_6 - 1.$$

(4) (6)

$G \text{ ó}$

$u(\rho, \varphi)$

$$u(y) \quad n, m \sim 15-20 \quad (\sim 10^{-9} - 10^{-11})$$

$$F(x)u \quad G(x)u.$$

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$\alpha \quad \beta$

$$F(x) \quad G(x).$$

[6]:

$$f_{nj}^1(x) = h_{nj} r^{n-2j} P_j^{n-2j}(2r^2 - 1) u_{n-2j}^{(1)}(\phi), \quad 0 \leq 2j \leq n,$$

$$f_{nj}^2(x) = h_{nj} r^{n-2j} P_j^{n-2j}(2r^2 - 1) u_{n-2j}^{(2)}(\phi), \quad 0 \leq 2j \leq n-1, \quad (7)$$

$h_{nj}$  ó  $x_1 = r \cos \phi, x_2 = r \sin \phi, u_m^{(1)}(\phi) = \theta_m \cos \phi, \theta_0 = 1/\sqrt{2}, \theta_m = 1$   
 $m > 0, u_m^{(2)}(\phi) = \sin \phi.$   
 (7)

$$\iint_S \frac{(1-|y|)^\beta}{|x-y|^{2\mu}} \left\{ \begin{matrix} f_{nj}^1 \\ f_{nj}^2 \end{matrix} \right\} dS_y = \delta_{j\mu} r^{n-2j} \left\{ \begin{matrix} u_{n-2j}^{(1)}(\phi) \\ u_{n-2j}^{(2)}(\phi) \end{matrix} \right\} {}_2F_1(a, b; c; r^2), \quad (8)$$

$$\delta_{j\mu} = -\sin(\pi\mu) \frac{h_{nj} (-1)^j \Gamma(\beta + j + 1) \Gamma^2(\mu + 1) \Gamma(n - j - \mu)}{j!(n - 2j)! \Gamma(\beta + j + \mu + 2)}, \quad 0 < \mu < 1, \quad a = n - j - \mu,$$

$$b = -1 - j - \beta - \mu, \quad {}_2F_1(a, b; c; x) \quad (8) \quad (2).$$

$$\beta = -1/2 \quad \mu = 1/2, \quad (8)$$

$$F(x) \left\{ \begin{matrix} f_{nj}^{1, -1/2} \\ f_{nj}^{2, -1/2} \end{matrix} \right\} = \lambda_{jn} \left\{ \begin{matrix} f_{nj}^{1, -1/2}(x) \\ f_{nj}^{2, -1/2}(x) \end{matrix} \right\}, \quad (9)$$

$$\lambda_{jn} = \frac{\pi \Gamma(j + 1/2) \Gamma(-j + n + 1/2)}{j!(n - j)!}.$$

$$\beta = 1/2 \quad \mu = 3/2 \quad (5),$$

$$G(x) \left\{ \begin{matrix} f_{nj}^{1, 1/2} \\ f_{nj}^{2, 1/2} \end{matrix} \right\} = \gamma_{jn} \left\{ \begin{matrix} f_{nj}^{1, 1/2}(x) \\ f_{nj}^{2, 1/2}(x) \end{matrix} \right\}, \quad (10)$$

$$\gamma_{jn} = -\frac{4\pi \Gamma(j + 3/2) \Gamma(-j + n + 3/2)}{j!(n - j)!}.$$

(9) (10)

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2. ... .. // . . . . .
3. ... .. 1992. ó 253 .
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5. ... .. 1992. ó 5.6 . 27631.
6. ... .. 2003 ó 688 .
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