[illegible] 519.6
[illegible]
e-mail: kulynych@ipm.lviv.ua
[illegible]

[illegible]

[illegible]

[illegible] [illegible] [1,2]:

$$F(x)u \equiv \iint\limits_S \frac{\left(1-|y|\right)^{\beta} u(y)}{|x-y|} dS_y = f(x),$$
$$G(x)u \equiv f.p.\iint\limits_S \frac{\left(1-|y|\right)^{\alpha} u(y)}{|x-y|^3} dS_y = g(x) \qquad (1)$$

[illegible] $x=\{x_1,x_2\}$, $y=\{y_1,y_2\}$, S ї [illegible] $x \in S$, $\beta > -1, \alpha \geq 0$. [illegible] $f.p.$ [illegible]

[illegible]

[illegible] [illegible] (1) [illegible] $F(x)u$ [illegible] $G(x)u$.

[illegible] (1) [illegible] $u(y)$ [illegible] $y=\{\rho\cos\varphi, \rho\sin\varphi\}$, $x=\{r\cos\phi, r\sin\phi\}$ [illegible] $u(y)\equiv u(\rho,\varphi)$ [illegible] φ [illegible] m [illegible] $\tau_l = 2\pi l/m$, $l=\overline{0,m-1}$ [illegible] $p=[m/2]$. [illegible] $u(\rho,\tau_l)$ [illegible] ρ [illegible] $n-1$ [illegible] $\{\rho_k\}$, [illegible] $P_n^{(1,\beta)}(1-2\rho)$ [illegible] $P_n^{(1,\alpha)}(1-2\rho)$.

[illegible] (1) [illegible] $F(x)u$ [illegible]

$$F_{kj}(r,\phi;\tau_l) = \int_0^1\int_0^{2\pi} \frac{\overline{P}_k^{(\beta)}(\rho)\cos\left[j(\varphi-\tau_l)\right] d\rho d\varphi}{\left(\rho^2+r^2-2\rho r\cos(\varphi-\phi)\right)^{1/2}}, \qquad (2)$$

[illegible] $G(x)u$ ї [illegible]

$$G_{kj}(r,\phi;\tau_l) = f.p.\int_0^1\int_0^{2\pi} \frac{\overline{P}_k^{(\alpha)}(\rho)\cos\left[j(\varphi-\tau_l)\right] d\rho d\varphi}{\left(\rho^2+r^2-2\rho r\cos(\varphi-\phi)\right)^{3/2}}, \qquad (3)$$

[illegible] $\overline{P}_k^{(\gamma)}(\rho) = \rho(1-\rho)^{\gamma} P_k^{(1,\gamma)}(1-2\rho)$, $k=\overline{0,n-1}$, $j=\overline{0,p}$.

[illegible] (2) [illegible] [3] [illegible]

$$\int_0^{2\pi} \frac{\cos\left[j\left(\varphi-\tau_l\right)\right] d\varphi}{\sqrt{\rho^2+r^2-2\rho r\cos\left(\varphi-\phi\right)}} = \frac{\left(-1\right)^j 2\pi \mathrm{R}^j\left(\rho,r\right)}{\left(1/2\right)_j \sqrt{\left|\rho^2-r^2\right|}} \cos j\left(\phi-\tau_l\right).$$

[illegible] $\mathrm{R}^j\left(\rho,r\right)=\mathrm{P}^j_{-1/2}\left(\left(\rho^2+r^2\right)/\left|\rho^2-r^2\right|\right)$, $\mathrm{P}^\mu_\nu\left(z\right)$ – [illegible] $\left(a\right)_j$ – [illegible] $\mathrm{P}^\mu_\nu\left(z\right)$ [illegible] $-\infty$ [illegible] $+1$. [illegible] $\left|1-x\right|^\nu \mathrm{P}^\mu_\nu\left(\left(1+x\right)/\left|1-x\right|\right)$ [4], [illegible]

$$F_{kj}\left(r,\phi;\tau_l\right) = J\left(\phi,\tau\right) G^{33}_{66}\left(r^2 \,\middle|\, \begin{matrix} a_1,\cdots a_6 \\ b_1,\cdots b_6 \end{matrix}\right), \tag{4}$$

[illegible] $a_1=\left(1-k\right)/2, a_2=-k/2, a_3=\left(1-j\right)/2$, $a_4=\left(1+j\right)/2$, $a_5=\left(2+k+\beta\right)/2$, $a_6=\left(3+k+\beta\right)/2$, $b_1=j/2, b_2=1/2, b_3=1$, $b_4=-j/2, b_5=0, b_6=1/2$, $G^{33}_{66}\left(r^2 \,\middle|\, \begin{matrix} a_1,\cdots a_6 \\ b_1,\cdots b_6 \end{matrix}\right)$ – G – [illegible] $J\left(\phi,\tau\right)=2^{-\beta}\pi\Gamma\left(\beta+k+1\right)\cos j\left(\phi-\tau\right)/k!$.

[illegible] (3) [illegible] [5]

$$f.p.\iint_S \frac{f\left(y\right)}{\left|x-y\right|^3} dS_y = \Delta_x \iint_S \frac{f\left(y\right)}{\left|x-y\right|} dS_y, \tag{5}$$

[illegible] Δ_x - [illegible]

[illegible]

$$G_{nj}\left(r,\phi;\tau\right) = J\left(\phi,\tau\right)\begin{pmatrix} -j^2 G^{33}_{66}\left(r^2 \,\middle|\, \begin{matrix} a_1,\cdots a_6 \\ b_1,\cdots b_6 \end{matrix}\right) + \\ +4G^{34}_{77}\left(r^2 \,\middle|\, \begin{matrix} c_1,\cdots c_7 \\ d_1,\cdots d_7 \end{matrix}\right) \end{pmatrix}, \tag{6}$$

[illegible] $a_1=\left(1-j\right)/2$, $a_2=-n/2$, $a_3=\left(1-n\right)/2$, $a_4=\left(j+1\right)/2$, $a_5=\left(2+n+\alpha\right)/2$, $a_6=a_5+1/2$; $b_1=1/2$, $b_2=1$, $b_3=j/2$, $b_4=0$, $b_5=b_1$, $b_6=-b_3$; $c_1=-1$, $c_{i+1}=a_i-1$, $i=\overline{1,6}$; $d_1=d_4=-b_1$, $d_2=d_5=d_6=0$, $d_3=b_3-1, d_7=b_6-1$.

[illegible] (4) [illegible] (6) [illegible] G – [illegible] $u\left(\rho,\varphi\right)$ [illegible].

[illegible] $u\left(y\right)$ [illegible] $n,m\sim15-20$ [illegible] ($\sim10^{-9}-10^{-11}$) [illegible] $F\left(x\right)u$ [illegible] $G\left(x\right)u$. [illegible] (1) [illegible].

[illegible]. [illegible] (1) [illegible] α [illegible] β [illegible] $F\left(x\right)$ [illegible] $G\left(x\right)$.

[illegible] [6]:

$$\begin{aligned} f^{1,\beta}_{nj}\left(x\right) &= h^{\beta}_{nj} r^{n-2j} P^{\beta,n-2j}_j\left(2r^2-1\right) u^{(1)}_{n-2j}\left(\phi\right), 0\le 2j\le n, \\ f^{2,\beta}_{nj}\left(x\right) &= h^{\beta}_{nj} r^{n-2j} P^{\beta,n-2j}_j\left(2r^2-1\right) u^{(2)}_{n-2j}\left(\phi\right), 0\le 2j\le n-1, \end{aligned} \tag{7}$$

[illegible] h_{nj}^{β} [illegible], $x_1 = r\cos\phi$, $x_2 = r\sin\phi$, $u_m^{(1)}(\phi) = \theta_m \cos\phi$, $\theta_0 = 1/\sqrt{2}$, $\theta_m = 1$ [illegible] $m > 0$, $u_m^{(2)}(\phi) = \sin\phi$.

[illegible] (7) [illegible]

$$\iint\limits_S \frac{\left(1-|y|\right)^{\beta}}{|x-y|^{2\mu}} \begin{Bmatrix} f_{nj}^{1,\beta} \\ f_{nj}^{2,\beta} \end{Bmatrix} dS_y = \delta_{jn\mu} r^{n-2j} \begin{Bmatrix} u_{n-2j}^{(1)}(\phi) \\ u_{n-2j}^{(2)}(\phi) \end{Bmatrix} {}_2F_1\left(a,b;c;r^2\right), \qquad (8)$$

[illegible] $\delta_{jn\mu} = -\sin(\pi\mu)\dfrac{h_{nj}^{\beta}(-1)^j \Gamma(\beta+j+1)\Gamma^2(\mu+1)\Gamma(n-j-\mu)}{j!(n-2j)!\Gamma(\beta+j+\mu+2)}$, $0<\mu<1$, $a = n-j-\mu$, $b=-1-j-\beta-\mu$, ${}_2F_1(a,b;c;x)$ [illegible]

[illegible] (8) [illegible] (2). [illegible] $\beta = -1/2$ [illegible] $\mu = 1/2$, [illegible] (8) [illegible]:

$$F(x)\begin{Bmatrix} f_{nj}^{1,-1/2} \\ f_{nj}^{2,-1/2} \end{Bmatrix} = \lambda_{jn} \begin{Bmatrix} f_{nj}^{1,-1/2}(x) \\ f_{nj}^{2,-1/2}(x) \end{Bmatrix}, \qquad (9)$$

[illegible] $$\lambda_{jn} = \frac{\pi\Gamma(j+1/2)\Gamma(-j+n+1/2)}{j!(n-j)!}.$$

[illegible] $\beta = 1/2$ [illegible] $\mu = 3/2$ [illegible] (5), [illegible]

$$G(x)\begin{Bmatrix} f_{nj}^{1,1/2} \\ f_{nj}^{2,1/2} \end{Bmatrix} = \gamma_{jn} \begin{Bmatrix} f_{nj}^{1,1/2}(x) \\ f_{nj}^{2,1/2}(x) \end{Bmatrix}, \qquad (10)$$

[illegible] $$\gamma_{jn} = -\frac{4\pi\Gamma(j+3/2)\Gamma(-j+n+31/2)}{j!(n-j)!}.$$

[illegible] (9) [illegible] (10) [illegible] (1).

1. [illegible] // [illegible] 1993. [illegible] 33. [illegible] 2. [illegible] 313–318.
2. [illegible] 1992. [illegible] 253 [illegible]
3. [illegible] // [illegible] 1992. [illegible] 5. [illegible] 27–31.
4. [illegible] 3-[illegible] 3. [illegible], 2003 [illegible] 688 [illegible]
5. [illegible] 2009. [illegible] 3. [illegible] 36–43.
6. [illegible] 2009. [illegible] 12.- [illegible] 27–32.