

519.9

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$$A(\lambda)Z(\lambda) = B(\lambda),$$

- 1)  $A(\lambda)$  ó  $B(\lambda)$  ó
- 2)  $A(\lambda)$  ó  $B(\lambda)$

$$\inf_{\lambda \in F} \psi(Z(\lambda)), \tag{1}$$

$$Z(\lambda) = \{Z_1(\lambda), Z_2(\lambda), \dots, Z_n(\lambda)\}$$

$$\psi(\lambda) = \sqrt{\sum_{i=1}^n Z_i^2(\lambda)} \quad \psi(\lambda) = \sqrt{\sum_{i=1}^n (Z_i(\lambda) - z_i^0)^2},$$

$$z^0 = \{z_1^0, z_2^0, \dots, z_n^0\} \quad \lambda \in F$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \tag{8}$$

$$\frac{\partial V_x}{\partial t} = V_x \frac{\partial V_x}{\partial X} + V_y \frac{\partial V_y}{\partial Y} + V_z \frac{\partial V_z}{\partial Z} = -\frac{1}{\rho} \frac{\partial p}{\partial X} + v \left( \frac{\partial^2 V_x}{\partial X^2} + \frac{\partial^2 V_x}{\partial Y^2} + \frac{\partial^2 V_x}{\partial Z^2} \right) + g_x \tag{2}$$

$$\frac{\partial V_y}{\partial t} = V_x \frac{\partial V_y}{\partial X} + V_y \frac{\partial V_y}{\partial Y} + V_z \frac{\partial V_y}{\partial Z} = \frac{1}{\rho} \frac{\partial p}{\partial Y} + v \left( \frac{\partial^2 V_y}{\partial X^2} + \frac{\partial^2 V_y}{\partial Y^2} + \frac{\partial^2 V_y}{\partial Z^2} \right) + g_y \tag{3}$$

$$\frac{\partial V_z}{\partial t} = V_x \frac{\partial V_z}{\partial X} + V_y \frac{\partial V_z}{\partial Y} + V_z \frac{\partial V_z}{\partial Z} = \frac{1}{\rho} \frac{\partial p}{\partial Z} + v \left( \frac{\partial^2 V_z}{\partial X^2} + \frac{\partial^2 V_z}{\partial Y^2} + \frac{\partial^2 V_z}{\partial Z^2} \right) + g_z.$$

(2), (3)

; X, Y, Z ó

$$\left( \begin{matrix} V_x, V_y, V_z \\ p \end{matrix} \right), \quad \bar{V}(\bar{R})$$

$$\bar{R} = 0,$$

[7]

$$V_k(\bar{R}) = V_k(\bar{0}) + G_{km} X_m, \quad G_{km} \equiv \left( \frac{\partial V_k}{\partial X_m} \right)_{R=0},$$

$$G_{11} + G_{22} + G_{33} = 0,$$

(4)

$V_k, G_{km}$  ó  
 $X_2, X_3$ .

$X_1$ ,

$$V_k(\bar{0}) = 0 \quad (4)$$

$$G_{km} = 0$$

$$\|G_{km}\|$$

$$\|G_{km}\| = \|E_{km}\| + \|\Omega_{km}\|, \quad E_{km} = E_{mk} = \frac{1}{2}(G_{km} + G_{mk}), \quad \Omega_{km} = -\Omega_{mk} = \frac{1}{2}(G_{km} - G_{mk}).$$

1.

$$V_x = GY, \quad V_y = 0, \quad V_z = 0,$$

$$\|G_{km}\| = \begin{vmatrix} 0 & G & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|E_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ \frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|\Omega_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ -\frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.$$

G

2.

ó

$$V_x = \frac{1}{2}GY, \quad V_y = \frac{1}{2}GX, \quad V_z = 0.$$

$$\|G_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ \frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|E_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ \frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|\Omega_{km}\| = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.$$

$q(x)$

: [2]

$$f(x) = \int_0^l G(x,s)q(s)ds, \tag{5}$$

$G(x,s)$  ó

( ),

$x$

$s$ .

$$q(x) \quad -m(s)\frac{\partial^2 f(s,t)}{\partial t^2},$$

$$m(s) = \frac{q(s)}{g}. \tag{5}$$

$$f(x,t) = -\int_0^l G(x,s)m(s)\frac{\partial^2 f(s,t)}{\partial t^2}ds. \tag{6}$$

$K(x,s) = G(x,s)m(x,s)$ :

$$f(x) = \lambda \int_0^l K(x,s)f(s)ds. \tag{7}$$

$G(x,s)$

$K(x,s)$

$m(x) = const$

ø

(7)  $f(x) \neq 0$

$\lambda$ ,  $\lambda$   
(7).

ø  $f(x)$

$\lambda$ .

$$(7) \quad i = x_i \in (0,l), \quad i = \overline{1,N}$$

$$\sum_{k=1}^N \sum_{j=1}^N K(\lambda, x_k, x_j) f_j(x_j) = f_k(x_k), \tag{8}$$

$K(\lambda, x_k, x_j)$  ó

$$\left. \begin{aligned} K(\lambda, x_1, x_1) &= \lambda G(x_1, x_1) m(x_1) \\ K(\lambda, x_1, x_2) &= \lambda G(x_1, x_2) m(x_2) \\ \dots \dots \dots \\ K(\lambda, x_1, x_n) &= \lambda G(x_1, x_n) m(x_n) \\ \dots \dots \dots \\ K(\lambda, x_n, x_1) &= \lambda G(x_n, x_1) m(x_1) \\ \dots \dots \dots \\ K(\lambda, x_n, x_2) &= \lambda G(x_n, x_2) m(x_2) \\ \dots \dots \dots \\ K(\lambda, x_n, x_n) &= \lambda G(x_n, x_n) m(x_n) \end{aligned} \right\}. \tag{9}$$

(7)  $m(x) = const, K(x, s)$  (8) ó

(10)  $D(\lambda) = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{a_n(\lambda)}{n!}$  (10)

(7).  $a_n(\lambda)$  (8),  
 $a_n(\lambda) = \int_0^1 \int_0^1 \dots \int_0^1 \begin{vmatrix} K(\lambda, x_1, x_1) & K(\lambda, x_1, x_2) & \dots & K(\lambda, x_1, x_n) \\ K(\lambda, x_n, x_1) & K(\lambda, x_n, x_2) & \dots & K(\lambda, x_n, x_n) \end{vmatrix} dx_1 \dots dx_n,$  (11)

$D(\lambda) = 0.$

(5)  $m(x) = const.$  (7)

$a_n(\lambda)$   $D(\lambda) = 0,$   $\lambda = p^2,$  (6)

$p$  ó  $n -$

(6)  $1 - a_1 \lambda = 0,$  (12)

(13)  $p_1 = \sqrt{\lambda} = \sqrt{\frac{1}{a_1}}$  (13)

(6)  $1 - a_1 \lambda + \frac{a_2}{2} \lambda^2 = 0.$  (14)

(14)  $\lambda_{1,2} = \frac{a_1 \pm \sqrt{a_1^2 - 2a_2}}{a_2}$

$p_1 = \sqrt{\frac{a_1 - \sqrt{a_1^2 - 2a_2}}{a_2}},$  (15)

$p_2 = \sqrt{\frac{a_1 + \sqrt{a_1^2 - 2a_2}}{a_2}},$  (16)

$p_1$  ó  $p_2$  ó

$$1 - a_1 \lambda + \frac{a_2}{2} \lambda^2 - \frac{a_3}{6} \lambda^3 = 0.$$

$n = 1, 2, \dots$

(6)

$m(x) = \text{const}$

$$G(\bar{x}, \bar{s}) = \frac{l^3}{EI} \int_0^{\bar{s}} (\bar{s} - u)(\bar{x} - u) du \quad \bar{x} \geq \bar{s}, \tag{17}$$

$\bar{x} = \frac{x}{l}$   
 $, l \text{ ó}$

(17)

$$G(\bar{x}, \bar{s}) = \frac{l^3}{6EI} \bar{s}^{-2} (3\bar{x} - \bar{s}) \quad \bar{x} \geq \bar{s}.$$

$$G(\bar{x}, \bar{s}) = \frac{l^3}{6EI} \bar{x}^{-2} (3\bar{s} - \bar{x}) \quad \bar{x} \leq \bar{s}.$$

$a_1 \quad a_2.$

(7)

(8)

$$a_1 = l \int_0^1 K(\bar{x}, \bar{x}) d\bar{x} = ml \int_0^1 G(\bar{x}, \bar{x}) d\bar{x} = \frac{ml^4}{3EI} \int_0^1 \bar{x}^{-3} d\bar{x}.$$

$$a_1 = \frac{1}{12} \frac{ml^4}{EI}. \tag{18}$$

(7)

(8),

$$a_2 = l^2 \int_0^1 \int_0^1 \begin{vmatrix} K(\bar{x}, \bar{x}) & K(\bar{x}, \bar{s}) \\ K(\bar{s}, \bar{x}) & K(\bar{s}, \bar{s}) \end{vmatrix} d\bar{x} d\bar{s} =$$

$$m^2 l^2 \int_0^1 \int_0^1 G(\bar{x}, \bar{x}) G(\bar{s}, \bar{s}) d\bar{x} d\bar{s} - m^2 l^2 \int_0^1 \int_0^1 G(\bar{x}, \bar{s}) G(\bar{x}, \bar{s}) d\bar{x} d\bar{s} =$$

$$= m^2 l^2 \left[ \int_0^1 G(\bar{x}, \bar{x}) d\bar{x} \right]^2 - m^2 l^2 \int_0^1 \int_0^1 [G(\bar{x}, \bar{s})]^2 d\bar{x} d\bar{s} =$$

$$= \frac{m^2 l^8}{9(EI)^2} \left[ \int_0^1 \bar{x}^{-2} d\bar{x} \right]^2 - \frac{m^2 l^8}{36(EI)^2} \int_0^1 \left\{ \int_0^{\bar{x}} \bar{s}^{-4} (3\bar{x} - \bar{s})^2 d\bar{s} + \int_{\bar{x}}^1 \bar{x}^{-2} (3\bar{s} - \bar{x})^2 d\bar{s} \right\} d\bar{x}.$$

$$a_2 = \frac{m^2 l^8}{2520(EI)^2}. \tag{19}$$

(18)

(13)

$$p_1 = \frac{3,46}{l^2} \sqrt{\frac{EI}{m}}. \tag{20}$$

(18) (19)

(15) (16)

"  $\emptyset$  - : , "

$$P_1 = \frac{3,516}{l^2} \sqrt{\frac{EI}{m}}, P_2 = \frac{20,19}{l^2} \sqrt{\frac{EI}{m}}. \quad (21)$$

$\emptyset$  . -  $\emptyset$  ,  
 $\emptyset$  [9],  
 $\emptyset$  ,  $\emptyset$   $n$  (  $\emptyset$  )  
 $\emptyset$  .

1. . . . . / . . . . , . . . . ,
2. . . . .  $\hat{\delta}$  . : , 1972.  $\hat{\delta}$  416 . / . . . . , . . . .
3.  $\hat{\delta}$  . : , 1965.  $\hat{\delta}$  526 . / . . . . // .  $\hat{\delta}$  1986.  $\hat{\delta}$  .41,
4. 1.  $\hat{\delta}$  . 171  $\hat{\delta}$  178. / . . . .  $\hat{\delta}$  . : , 1989.
5.  $\hat{\delta}$  448 . / . . . .  $\hat{\delta}$  . : , 1961.  $\hat{\delta}$  308 .
6. . . . . / . . . .  $\hat{\delta}$  . : , 1974.  $\hat{\delta}$  257 .
7. . . . . / . . . . , . . . .  $\hat{\delta}$  . : , 1977.  $\hat{\delta}$  352 .
8. . . . . / . . . . , . . . .  $\hat{\delta}$  . : , 1986.  $\hat{\delta}$  736 .
9. . . . .  $\lambda$ - / . . . . , . . . .  $\hat{\delta}$  . : , 2007.  $\hat{\delta}$  294 .
10. // . . . .  $\hat{\delta}$  1996.  $\hat{\delta}$  2.  $\hat{\delta}$  . 120  $\hat{\delta}$  131. / . . . .
11.  $\hat{\delta}$  . : , 1989.  $\hat{\delta}$  184 . / . . . .