

519.9

olhakov@gmail.com, chip.ukraine@gmail.com.

$$A(\lambda)Z(\lambda) = B(\lambda),$$

- 1) $A(\lambda)$ $B(\lambda)$ ó
- 2) $A(\lambda)$ ó $B(\lambda)$

$$\inf_{\lambda \in F} \psi(Z(\lambda)), \tag{1}$$

$$Z(\lambda) = \{Z_1(\lambda), Z_2(\lambda), \dots, Z_n(\lambda)\}$$

$$\psi(\lambda) = \sqrt{\sum_{i=1}^n Z_i^2(\lambda)} \quad \psi(\lambda) = \sqrt{\sum_{i=1}^n (Z_i(\lambda) - z_i^0)^2},$$

$$z^0 = \{z_1^0, z_2^0, \dots, z_n^0\} \quad \lambda \in F$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \tag{8}$$

$$\frac{\partial V_x}{\partial t} = V_x \frac{\partial V_x}{\partial X} + V_y \frac{\partial V_y}{\partial Y} + V_z \frac{\partial V_z}{\partial Z} = -\frac{1}{\rho} \frac{\partial p}{\partial X} + v \left(\frac{\partial^2 V_x}{\partial X^2} + \frac{\partial^2 V_x}{\partial Y^2} + \frac{\partial^2 V_x}{\partial Z^2} \right) + g_x \tag{2}$$

$$\frac{\partial V_y}{\partial t} = V_x \frac{\partial V_y}{\partial X} + V_y \frac{\partial V_y}{\partial Y} + V_z \frac{\partial V_y}{\partial Z} = \frac{1}{\rho} \frac{\partial p}{\partial Y} + v \left(\frac{\partial^2 V_y}{\partial X^2} + \frac{\partial^2 V_y}{\partial Y^2} + \frac{\partial^2 V_y}{\partial Z^2} \right) + g_y \tag{3}$$

$$\frac{\partial V_z}{\partial t} = V_x \frac{\partial V_z}{\partial X} + V_y \frac{\partial V_z}{\partial Y} + V_z \frac{\partial V_z}{\partial Z} = \frac{1}{\rho} \frac{\partial p}{\partial Z} + v \left(\frac{\partial^2 V_z}{\partial X^2} + \frac{\partial^2 V_z}{\partial Y^2} + \frac{\partial^2 V_z}{\partial Z^2} \right) + g_z.$$

(2), (3) ; X, Y, Z ó

$$(V_x, V_y, V_z), \quad g_x, g_y, g_z \quad p.$$

$$\bar{R} = 0,$$

[7]

$$V_k(\bar{R}) = V_k(\bar{0}) + G_{km} X_m, \quad G_{km} \equiv \left(\frac{\partial V_k}{\partial X_m} \right)_{R=0},$$

$$G_{11} + G_{22} + G_{33} = 0, \tag{4}$$

V_k, G_{km} ó X_1, X_2, X_3 .

$$V_k(\bar{0}) = 0 \tag{4}$$

$$G_{km} = 0$$

$$\|G_{km}\|$$

$$\|G_{km}\| = \|E_{km}\| + \|\Omega_{km}\|, \quad E_{km} = E_{mk} = \frac{1}{2}(G_{km} + G_{mk}), \quad \Omega_{km} = -\Omega_{mk} = \frac{1}{2}(G_{km} - G_{mk}).$$

1.

$$V_x = GY, \quad V_y = 0, \quad V_z = 0,$$

$$\|G_{km}\| = \begin{vmatrix} 0 & G & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|E_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ \frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|\Omega_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ -\frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.$$

G

2.

$$V_x = \frac{1}{2}GY, \quad V_y = \frac{1}{2}GX, \quad V_z = 0.$$

$$\|G_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ \frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|E_{km}\| = \begin{vmatrix} 0 & \frac{1}{2}G & 0 \\ \frac{1}{2}G & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \|\Omega_{km}\| = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.$$

, 2012.

9

$q(x)$

: [2]

$$f(x) = \int_0^l G(x,s)q(s)ds, \tag{5}$$

$G(x,s)$ ó

(),

x

s .

$$q(x) \quad -m(s)\frac{\partial^2 f(s,t)}{\partial t^2},$$

$$m(s) = \frac{q(s)}{g}. \tag{5}$$

$$f(x,t) = -\int_0^l G(x,s)m(s)\frac{\partial^2 f(s,t)}{\partial t^2}ds. \tag{6}$$

$K(x,s) = G(x,s)m(x,s)$:

$$f(x) = \lambda \int_0^l K(x,s)f(s)ds. \tag{7}$$

$G(x,s)$

$K(x,s)$

$m(x) = const$

ø

(7) $f(x) \neq 0$

λ, λ

(7).

ø $f(x)$

λ .

(7)

$$i = x_i \in (0,l), i = \overline{1,N}$$

$$\sum_{k=1}^N \sum_{j=1}^N K(\lambda, x_k, x_j) f_j(x_j) = f_k(x_k), \tag{8}$$

$K(\lambda, x_k, x_j)$ ó

$$\left. \begin{aligned} K(\lambda, x_1, x_1) &= \lambda G(x_1, x_1) m(x_1) \\ K(\lambda, x_1, x_2) &= \lambda G(x_1, x_2) m(x_2) \\ \dots \dots \dots \\ K(\lambda, x_1, x_n) &= \lambda G(x_1, x_n) m(x_n) \\ \dots \dots \dots \\ K(\lambda, x_n, x_1) &= \lambda G(x_n, x_1) m(x_1) \\ \dots \dots \dots \\ K(\lambda, x_n, x_2) &= \lambda G(x_n, x_2) m(x_2) \\ \dots \dots \dots \\ K(\lambda, x_n, x_n) &= \lambda G(x_n, x_n) m(x_n) \end{aligned} \right\}. \tag{9}$$

(7) $m(x) = \text{const}, \quad K(x, s)$ (8) ó

(10) $D(\lambda) = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{a_n(\lambda)}{n!}$. (10)

(7). $a_n(\lambda)$ (8),

$$a_n(\lambda) = \int_0^1 \int_0^1 \left| \begin{matrix} K(\lambda, x_1, x_1) & K(\lambda, x_1, x_2) & \dots & K(\lambda, x_1, x_n) \\ K(\lambda, x_n, x_1) & K(\lambda, x_n, x_2) & \dots & K(\lambda, x_n, x_n) \end{matrix} \right| dx_1 \dots dx_n, \quad (11)$$

(11) (11) (9). (10)

$D(\lambda) = 0.$

(5) $m(x) = \text{const}.$ (7)

$a_n(\lambda)$ $D(\lambda) = 0,$ $\lambda = p^2,$ (6)

p ó $n -$

(6) $1 - a_1 \lambda = 0,$ (12)

(13) $p_1 = \sqrt{\lambda} = \sqrt{\frac{1}{a_1}}.$ (13)

(6) $1 - a_1 \lambda + \frac{a_2}{2} \lambda^2 = 0.$ (14)

(14) $\lambda_{1,2} = \frac{a_1 \pm \sqrt{a_1^2 - 2a_2}}{a_2}$

$p_1 = \sqrt{\frac{a_1 - \sqrt{a_1^2 - 2a_2}}{a_2}},$ (15)

$p_2 = \sqrt{\frac{a_1 + \sqrt{a_1^2 - 2a_2}}{a_2}},$ (16)

p_1 ó p_2 ó

$$1 - a_1 \lambda + \frac{a_2}{2} \lambda^2 - \frac{a_3}{6} \lambda^3 = 0.$$

$n = 1, 2, \dots$

(6)

$m(x) = const$

$$G(\bar{x}, \bar{s}) = \frac{l^3}{EI} \int_0^{\bar{s}} (\bar{s} - u)(\bar{x} - u) du \quad \bar{x} \geq \bar{s}, \tag{17}$$

$\bar{x} = \frac{x}{l}$
 $, l \text{ ó}$

(17)

$$G(\bar{x}, \bar{s}) = \frac{l^3}{6EI} \bar{s}^{-2} (3\bar{x} - \bar{s}) \quad \bar{x} \geq \bar{s}.$$

$$G(\bar{x}, \bar{s}) = \frac{l^3}{6EI} \bar{x}^{-2} (3\bar{s} - \bar{x}) \quad \bar{x} \leq \bar{s}.$$

$a_1 \quad a_2.$

(7)

(8)

$$a_1 = l \int_0^1 K(\bar{x}, \bar{x}) d\bar{x} = ml \int_0^1 G(\bar{x}, \bar{x}) d\bar{x} = \frac{ml^4}{3EI} \int_0^1 \bar{x}^{-3} d\bar{x}.$$

$$a_1 = \frac{1}{12} \frac{ml^4}{EI}. \tag{18}$$

(7)

(8),

$$a_2 = l^2 \int_0^1 \int_0^1 \begin{vmatrix} K(\bar{x}, \bar{x}) & K(\bar{x}, \bar{s}) \\ K(\bar{s}, \bar{x}) & K(\bar{s}, \bar{s}) \end{vmatrix} d\bar{x} d\bar{s} =$$

$$m^2 l^2 \int_0^1 \int_0^1 G(\bar{x}, \bar{x}) G(\bar{s}, \bar{s}) d\bar{x} d\bar{s} - m^2 l^2 \int_0^1 \int_0^1 G(\bar{x}, \bar{s}) G(\bar{x}, \bar{s}) d\bar{x} d\bar{s} =$$

$$= m^2 l^2 \left[\int_0^1 G(\bar{x}, \bar{x}) d\bar{x} \right]^2 - m^2 l^2 \int_0^1 \int_0^1 [G(\bar{x}, \bar{s})]^2 d\bar{x} d\bar{s} =$$

$$= \frac{m^2 l^8}{9(EI)^2} \left[\int_0^1 \bar{x}^{-2} d\bar{x} \right]^2 - \frac{m^2 l^8}{36(EI)^2} \int_0^1 \left\{ \int_0^{\bar{x}} \bar{s}^{-4} (3\bar{x} - \bar{s})^2 d\bar{s} + \int_{\bar{x}}^1 \bar{x}^{-2} (3\bar{s} - \bar{x})^2 d\bar{s} \right\} d\bar{x}.$$

$$a_2 = \frac{m^2 l^8}{2520(EI)^2}. \tag{19}$$

(18)

(13)

$$p_1 = \frac{3,46}{l^2} \sqrt{\frac{EI}{m}}. \tag{20}$$

(18) (19)

(15) (16)

" \emptyset - : , "

$$P_1 = \frac{3,516}{l^2} \sqrt{\frac{EI}{m}}, P_2 = \frac{20,19}{l^2} \sqrt{\frac{EI}{m}}. \quad (21)$$

\emptyset . - \emptyset ,
 \emptyset [9],
 \emptyset , \emptyset n (\emptyset)
 \emptyset .

1. / , ,
2. $\hat{\delta}$. : , 1972. $\hat{\delta}$ 416 . / ,
3. $\hat{\delta}$. : , 1965. $\hat{\delta}$ 526 . / // . $\hat{\delta}$ 1986. $\hat{\delta}$.41,
4. 1. $\hat{\delta}$. 171 $\hat{\delta}$ 178. / $\hat{\delta}$. : , 1989.
5. $\hat{\delta}$ 448 . / $\hat{\delta}$. : , 1961. $\hat{\delta}$ 308 .
6. / $\hat{\delta}$. : , 1974. $\hat{\delta}$ 257 .
7. / , $\hat{\delta}$. : , 1977. $\hat{\delta}$ 352 .
8. / , $\hat{\delta}$. : , 1986. $\hat{\delta}$ 736 .
9. λ - / , $\hat{\delta}$. : , 2007. $\hat{\delta}$ 294 .
10. // $\hat{\delta}$ 1996. $\hat{\delta}$ 2. $\hat{\delta}$. 120 $\hat{\delta}$ 131. /
11. $\hat{\delta}$. : , 1989. $\hat{\delta}$ 184 . /